

## Autonomous vehicle tracking control for a curved trajectory

**Hasnawiya Hasan<sup>1,2</sup>, Faizal Arya Samman<sup>1</sup>, Muh Anshar<sup>1</sup>, Rhiza S. Sadjad<sup>1</sup>**

<sup>1</sup>Department of Electrical Engineering, Faculty of Engineering, Hasanuddin University, Makassar, Indonesia

<sup>2</sup>Department of Marine System Engineering, Faculty of Engineering, Hasanuddin University, Makassar, Indonesia

---

### Article Info

#### Article history:

Received Feb 20, 2023

Revised Sep 18, 2023

Accepted Nov 14, 2023

#### Keywords:

Autonomous vehicle  
Linear-quadratic regulator  
Observer  
State feedback linearization  
Tracking control

---

### ABSTRACT

Recently, research about trajectory tracking of autonomous vehicles has significantly contributed to the development of autonomous vehicle technology, particularly with novel control methods. However, tracking a curved trajectory is still a challenge for autonomous vehicles. This research proposes a state feedback linearization with observer feedback to overcome some difficulties arising from such a path. This approach suits a complex nonlinear system such as an autonomous vehicle. This method also has been compared with the linear-quadratic regulator (LQR) method. So, the goal of this research is to improve the control system performance of autonomous vehicles that are stable enough to navigate a curved path. Moreover, the study shows that the developed control law can track the curved path and solve existing problems. However, improvements are still necessary for the vehicle's performance and robustness.

*This is an open access article under the [CC BY-SA](#) license.*



---

### Corresponding Author:

Faizal Arya Samman

Department of Electrical Engineering, Faculty of Engineering Hasanuddin University

Jl. Poros Malino Km 6, Gowa, Makassar, South Sulawesi, Indonesia

Email: faizalas@unhas.ac.id

---

## 1. INTRODUCTION

Autonomous vehicle innovation has reached a milestone of achievements in the last few years. The control of a vehicle's motion is a critical part of autonomous vehicles, as it directly impacts the vehicle's safety and the satisfaction of its passengers. So, it is crucial to develop a path-following controller that overcomes problems in challenging environments [1].

The majority of current motion control research focuses on normal conditions. It is necessary to expand the research into difficult working conditions, such as curved paths, to achieve the promise of autonomous vehicles in managing crucial scenarios that human drivers find difficult or are unable to handle. Therefore, nonlinearity characteristics and multi-dimensional coupled dynamics are necessary to be improved under difficult operating conditions. Moreover, the requirements of system modeling, robustness, and adaptability of motion control algorithms are now much higher. In addition, research on the integration of motion planning and control which considers environmental parameters also is required to deal with multi-objective coordination in complicated scenarios [2]. So, research about tracking control is a demanding topic amount of other research topics on autonomous vehicle technology [3], [4].

Previous research on tracking control of autonomous vehicles using pure pursuit and stanley control shows satisfactory performance, especially in position control, and it has great benefits due to its simple design [5]. Park and Han [6] studied an adaptive pure pursuit controller to regulate the model, however, the data parameter is difficult to obtain and necessary to be reimplemented every time the platform is changed [7]. The study also demonstrates positive results for low-speed vehicle and linear road designs. However, this control approach cannot meet the requirements for driving on large road curvature and at high speed [5].

Proportional integral derivative (PID) simplicity design is a tremendous advantage for engineering applications. PID is also familiar with tracking control problems [5]. However, its poor versatility and parameter tuning are unsuitable for dynamic systems and environments. Fuzzy PID is also used for autonomous vehicle control [8], [9]. Even though the simulation results show stable tracking performance, the system still requires optimization. However, the tuning process is more complicated.

Linear-quadratic regulator (LQR) is a conventional optimal control approach, which develops an optimal controller using state feedback, this method is simple and frequently used in vehicle tracking control [5]. Hu *et al.* [10] developed an LQR control approach combining the future path's tangent direction and the heading angle. However, they still cannot ensure the system's stability when improving the path following task. Moreover, the LQR route tracking control approach based on visual road detection was also studied by Zhang and Zhu [11]. However, the research has not experimented with road tests [10]. Furthermore, some previous studies also have developed a less complex vehicle and presented control algorithms based on the LQR approach [6], [12], [13]. Therefore, a dynamic and a state space model were used to determine the LQR controller. So that improved vehicle performance can be reached while the vehicle is operated at a low speed on simple terrain. However, the controller's effects significantly decrease because of linear feedback and model simplification when the vehicle performs nonlinearly due to modeling errors and disturbances. Therefore they added feedforward control based on road information [12], [14] and feedback control based on vehicle dynamics to the controller design [12]-[15]. However, this controller must be adjusted online, which requires powerful processing and the controllers' design depends on the linear assumption, which limits their use.

The feedforward controller is developed based on feedforward data, i.e., road curvature and steering characteristics [5]. The autonomous vehicle has a large amount of I/O data. This large amount of data is used to develop model-free adaptive control to solve deviation angle tracking problems [16]. This approach has the benefit of a simple layout controller. However, this method is well-known as a black box, and it is not easy to analyze its stability and the high installation cost due to the expensive sensors [5].

Model predictive control (MPC) can cope with future prediction and system constraints in design development. It minimizes the error of the control design in a prediction horizon. Therefore, Sun *et al.* [17] introduced a new MPC control law and switched tracking errors. However, the approach only was experimented with in one path scenario [17]. The work conducted by Dai *et al.* [18] suggests an MPC controller and adaptive preview properties and a new longitudinal vehicle control. However, the speed control still cannot cope with the constraint. Furthermore, some previous researchers often simplified dynamics models and linearization to improve the efficiency of MPC. Therefore, some data are lost, and modeling errors occur during the design process. Thus, the vehicle cannot operate on large curvature roads and high-speed driving. This controller can generate steering control input and longitudinal tire force. However, this controller still necessary to be improved in real-time conditions, poor computation, and trajectory tracking stability analysis still require improvement [5].

Trajectory tracking using a state feedback linearization approach suitable for a class of nonlinear systems such as autonomous vehicles [19], [20]. This method is a popular approach among other control system design methods. The goal is to achieve good performance and stable conditions at the equilibrium [21], [22]. This approach also has a simple computation but is appropriate to apply in a highly nonlinear model, i.e., an autonomous vehicle [23]. Moreover, control algorithms for autonomous vehicles must meet the standard of safety and robustness, particularly in complex environments [23], [24].

Keeping the autonomous vehicle stable and robust in tracking predefined trajectories is essential in autonomous control system design. The real tracking field is sometimes hard to track, i.e., a curvature road. Drifting sometimes occurs when tracking a sharp bend at a high speed, so the vehicle requires to drive at a low speed when tracking this path [25]-[27]. The feedback linearization approach is appropriate to implement in a complex road scenario such as a curvature road because it considers all parameters and conditions from the system model and its environments to meet the requirement of a dynamic system's stable and robust condition [28], [29].

The benefit of the state feedback linearization approach using observer feedback is that it can overcome the unpredictable parameters and disturbances that typically result from nonlinear models, enhancing advantages and ensuring system stability. Observer feedback in this method could minimize the need for sensors naturally added to monitor these uncertain characteristics. The observer feedback can substitute these sensors in a complex system so that the system can limit the number of sensors that are used in the methods and simplify the architecture [30], [31]. Other approaches usually require many sensors to measure these parameters.

Furthermore, this research goal is to meet the requirement of a stable and robust control system and apply theoretical control approaches such as state feedback linearization and observer feedback into autonomous vehicle technology. Therefore, the paper is prepared as follows; section 2 is about developing a

control law using the full-state feedback linearization approach and adding observer feedback to design new estimated states. In section 2, the full states feedback linearization with observer based on LQR is also designed for comparison. Section 3 discusses the research result of this method using the simulation software MATLAB. Finally, section 4 summarizes all the research progress and results.

## 2. CONTROL LAW DESIGN

### 2.1. State feedback linearization with observer feedback

This research applies a state feedback linearization and an observer feedback approach for tracking control of the autonomous vehicle along curved path such as circular and sinusoidal paths. Observer feedback is designed to estimate more states from the model and optimize its system's output. So, the autonomous vehicle model chosen in this research is a kinematic model that represents motion regardless of the vehicle's dynamic components, such as force, torque, and inertia effects [32], [33]. The vehicle kinematic model is presented in Figure 1 as a two-dimensional coordinate (X-Y). Slipping and inertia parameters are not considered in this system [31]. The kinematic formula is represented as (1)-(3):

$$\dot{x} = v_x \cos \theta \quad (1)$$

$$\dot{y} = v_x \sin \theta \quad (2)$$

$$\dot{\theta} = \frac{v_x}{L} \tan \alpha \quad (3)$$

Where (x,y) represents the vehicle position on the ground based on space coordinates,  $\theta \in (0,2 \pi)$  is an azimuth angle that also corresponds to the vehicle position on the ground, and L is the length of the vehicle base. While the control inputs are u corresponds to the azimuth angle ( $\Theta$ ) and steering angle ( $\alpha$ ). This model is for low-speed and acceleration vehicles [32], [33].

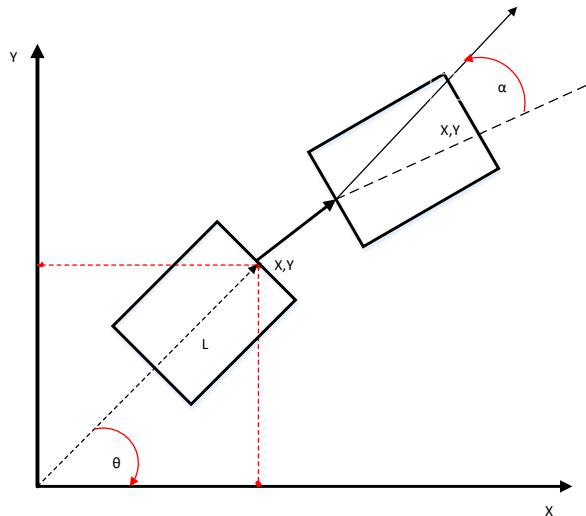


Figure 1. Vehicle model (top view) in 2D (X-Y coordinates)

The vehicle kinematic model is transformed into state vectors, i.e.:

$$dq = [\dot{x}, \dot{y}, \dot{\theta}] \quad (4)$$

$$\dot{x}_1 = x_2 \quad (5)$$

$$\dot{x}_2 = v_x \cos \theta = u(1) \quad (6)$$

$$\dot{y}_1 = y_2 \quad (7)$$

$$\dot{y}_2 = v_x \sin \theta = u(2) \quad (8)$$

$$\dot{\theta}_1 = \theta_2 \quad (9)$$

$$\dot{\theta}_2 = \frac{v_x}{L} \tan \alpha \quad (10)$$

Where  $v_x$  corresponds to the vehicle's velocity,  $(X, Y)$  is the vehicle's position on the ground,  $\Theta$  is the heading angle of the vehicle,  $L$  is the vehicle base's length, and  $\alpha$  is the vehicle's steering angle. Then, (5)-(10) can be transformed into a state vector form, such as (11)-(16):

$$\dot{x} = A x + B u \quad (11)$$

$$C_x = C x \quad (12)$$

$$\dot{y} = A y + B u \quad (13)$$

$$C_y = C y \quad (14)$$

$$\dot{\theta} = A \theta + B u \quad (15)$$

$$C_\theta = C \theta \quad (16)$$

$$\text{With } \dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}; x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; \dot{y} = \begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix}; y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}; \dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}; A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; C = [0 \quad 1]$$

Then, control feedback is designed to meet the control law requirement and consider the input reference such as (17):

$$u_x = -K \cdot x + r_x \quad (17)$$

$K = [K_1, K_2]$  is the state feedback matrix, and  $r_x$  is the reference input. Design a characteristic equation with the desired overshoot and natural frequency, such as (18):

$$\rho^2 + 2\sigma\varphi_n\rho + \varphi_n^2 = 0 \quad (18)$$

Where  $\sigma$  is overshoot and  $\varphi_n$  is a natural frequency.

The control law design is developed based on state feedback control method [32]. The state feedback gains are adjusted with this characteristic equation to reach stability around an equilibrium point. Then, observer feedback is added to this full-state feedback system, as represented in Figure 2. The new state vector is transformed to:

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} D_1 \\ D_2 \end{bmatrix} \hat{y} \quad (19)$$

$$\hat{y} = y - [0 \quad 1] \hat{x} \quad (20)$$

Where  $C = [0 \quad 1]$ , while  $\hat{x}$  corresponds to the state  $x$  estimation and  $D$  is the gain of the observer. This observer gain  $D$  must be observable to fulfill the system requirement. Assume the desired characteristic is given by:

$$\delta_d = \delta^2 + 2\tau\omega_n\delta + \omega_n^2 \quad (21)$$

Then, observer feedback  $D$  is computed to suit with desired overshoot ( $\tau = 0.5$ ) and natural frequency ( $\omega_n = 1$ ), i.e.

$$\det [\lambda I - (A - D \cdot C)] = 0 \quad (22)$$

$$\lambda^2 + D1\lambda + D2 = \delta^2 + \delta + 1 \quad (23)$$

The actual coordinates  $(x, y)$  related to desired coordinates  $(x_d, y_d)$  or reference input  $(r_x)$  are described as (24) and (25):

$$\dot{x} = \dot{x}_d + k_x x_e \quad (24)$$

$$\dot{y} = \dot{y}_d + k_y y_e \quad (25)$$

Where  $k_x$  and  $k_y$  are controller gain:

$$x_e = x_d - x \quad (26)$$

$$y_e = y_d - y \quad (27)$$

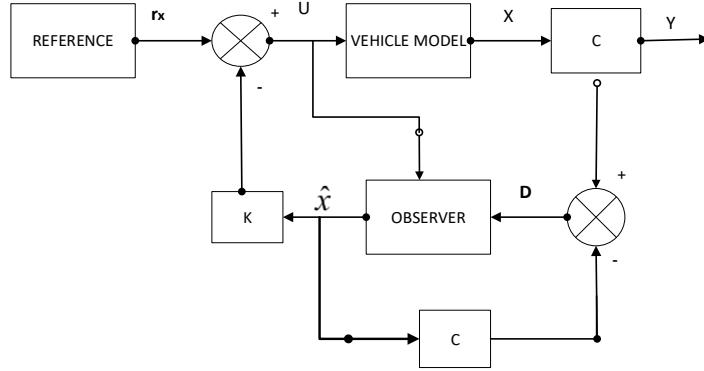


Figure 2. Control architecture of full state feedback linearization and observer feedback

Transform (18) and (19) into a state matrix as (28):

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \end{bmatrix} + \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix} \begin{bmatrix} x_e \\ y_e \end{bmatrix} = 0 \quad (28)$$

The following Lyapunov function, which is positive definite [20], guarantees the stability of the system with a distinct equilibrium at the origin:

$$V = \frac{1}{2} e^T e, e > 0 \quad (29)$$

where

$$e = \begin{bmatrix} x_e \\ y_e \end{bmatrix} \quad (30)$$

$$\dot{e} = - \begin{bmatrix} k_x, x_e \\ k_y, y_e \end{bmatrix} \quad (31)$$

and a derivative of Lyapunov function  $V$  is a negative definite [34], as (32):

$$\dot{V} = e^T \cdot \dot{e} = -k_x x_e^2 - k_y y_e^2 < 0 \quad (32)$$

The error of this function converges to zero, and the system is asymptotically stable. The new controller computes a steering angle to enable the vehicle to track the desired course.

## 2.2. Linear-quadratic regulator-based state variable feedback with full-observer design

In this section an LQR-based state variable feedback with full-observer is designed as a comparison with the proposed method in subsection 2.1. Assume that the system in (11) is controllable and all the states can be identified. Set  $x$  and  $u$  to a minimal value:

$$x^T x \rightarrow 0 \quad (33)$$

$$u^T u \rightarrow 0 \quad (34)$$

Both equations are always true, so the total cost function, i.e.:

$$J = (x^T x + u^T u) dt \quad (35)$$

Set some weighting Q and R to generalize function J, i.e.:

$$J = (x^T Q x + u^T R u) dt \quad (36)$$

Then minimize the cost function according to reference [35] until the Riccati equation is achieved, i.e.

$$A^T P + P A + P B R^{-1} B^T P + Q = 0 \quad (37)$$

The Riccati equation can be solved numerically to find matrix P.

Then design an observer based on LQR so that the (19) has an observer in the form:

$$\dot{\hat{x}} = A\hat{x} + Bu + D(y - Cx) \quad (38)$$

So, the observer error is

$$\dot{e} = \dot{x} - \dot{\hat{x}} = (A - DC)e \quad (39)$$

While the control feedback using estimates states is:

$$\dot{x} = Ax - BK\hat{x} \quad (40)$$

So, the full state space feedback is:

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - DC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \quad (41)$$

$$A - DC = A^T - C^T D^T \quad (42)$$

So that the observer design feedback is:

$$\dot{x} = A^T x + C^T u \quad (43)$$

$$u = -Dx \quad (44)$$

### 3. SIMULATION RESULTS AND DISCUSSION

#### 3.1. Full state feedback linearization and observer

An autonomous vehicle that tracks a linear and curved trajectory. The velocity is fixed at 10 m/s, starting at point (0,0) in X-Y coordinates. The trajectory control utilizes full state feedback linearization and observer. The simulation results as shown in Figures 3-5, which plots the actual trajectory and desired trajectory of a linear trajectory and curved trajectory.

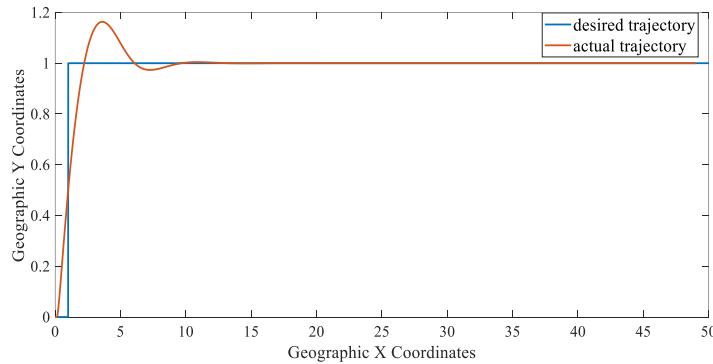


Figure 3. Linear trajectory tracking control using full state feedback and observer

The actual trajectory has a slight error difference from the desired trajectory that approves the developed method. The goal of this approach is to put the desired poles in the stable space of the system. However, the system requirement should be controllable to achieve the desired condition so that the controllable system has the poles in the stability domain [32].

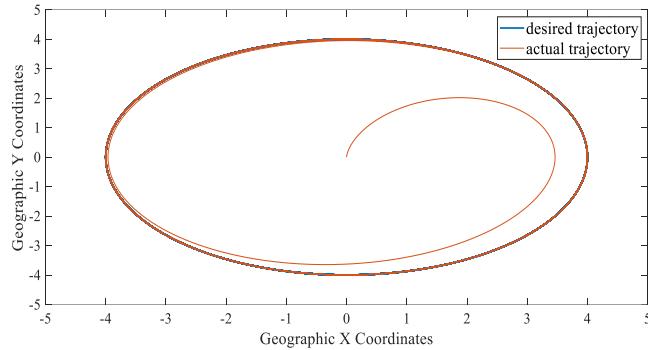


Figure 4. Circular trajectory tracking control using full state feedback and observer

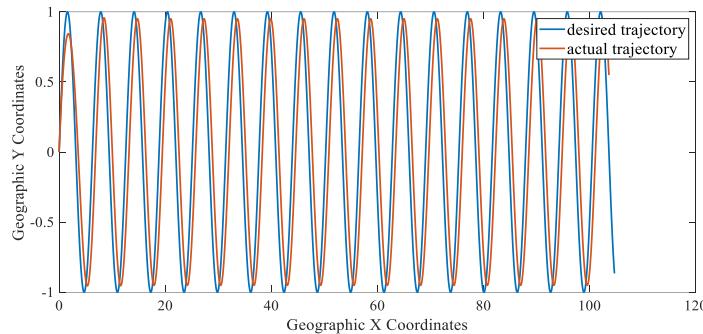


Figure 5. Sinusoidal trajectory tracking control using full state feedback and observer

In this autonomous vehicle system, the overshoot is 0,5, and natural frequency is 1, so the controller  $K$  is [1 1] and observer gain  $D$  is [1 1]. The vehicle is quite difficult to track the reference trajectory at a few seconds of the initial simulation time because the overshoot parameter had been set up. However, the vehicle response is stable according to the settling time setup.

Therefore, the overshoot parameter is set as low as possible to suit the desired settling time parameter and to have a faster response. The chosen controller is adjusted with the model's desired response. Control performance depends on the desired overshoot, desired settling time, and chosen controller parameter.

The ideal closed-loop control system has all states always available for feedback and a measurable process. However, only a few of the states of the system that available for feedback and measurement. So, the plan usually uses many sensors to read undetected states [32], which is expensive and complex. Therefore, this system designed an observer to reduce the number of sensors occupied.

The system should be observable to meet the requirements of this approach. The desired characteristic response works in the stable domain. While the lateral error exists because of the design of settling time and overshoot, however, this model was fast enough to achieve stability. The model achieves stability after a few seconds in accordance with the desired settling time and overshoot design. The trajectory error is shown in Figure 6. The lateral error can be computed using the Lyapunov method, as represented in Figure 6. The lateral error is adjusted to asymptotically stable. Therefore, the error always converges to zero.

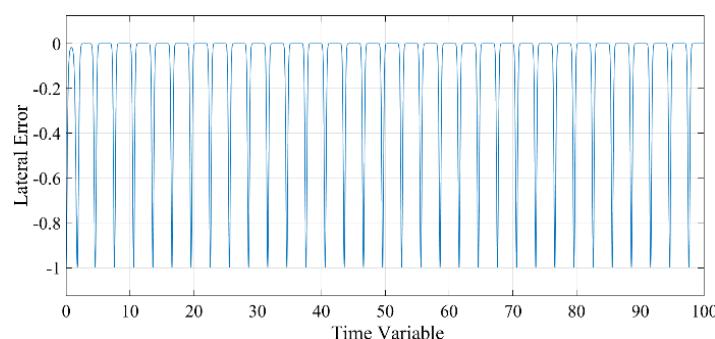


Figure 6. Trajectory error of sinusoidal trajectory

Moreover, Figure 7 represents the steering angle simulation results. The simulation result shows that at the initial condition, the controller tried to follow the predefined trajectory. However, the controller needs a few seconds to find the desired trajectory and reach stability. The steering angle is sinusoidal similar to the predefined trajectory because the steering angle tries to track the desired sinusoidal trajectory.

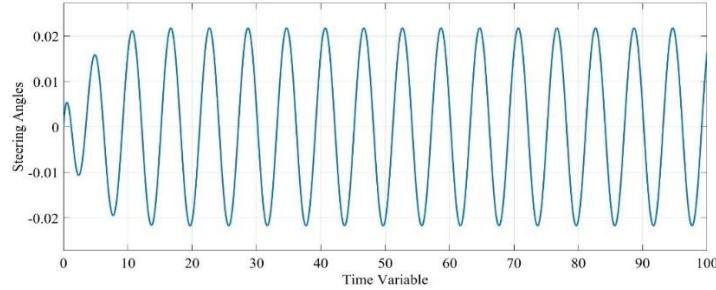


Figure 7. The steering angle which is generated from the sinusoidal trajectory

### 3.2. Linear-quadratic regulator-based state variable feedback with full-observer design

This approach is similar to the full state feedback linearization method. However, the gain controller  $K$  and gain observer design are based on LQR. Therefore, we choose gain controller  $K_{\text{optimum}} = [1 \ 1,7321]$  and gain observer  $D_{\text{optimum}} = [1,7321 \ 1]$ . While, the gain controller and gain observer in the previous approach, is based on overshoot and natural frequency requirement. Figures 8-10 shows the simulation results of this LQR approach.

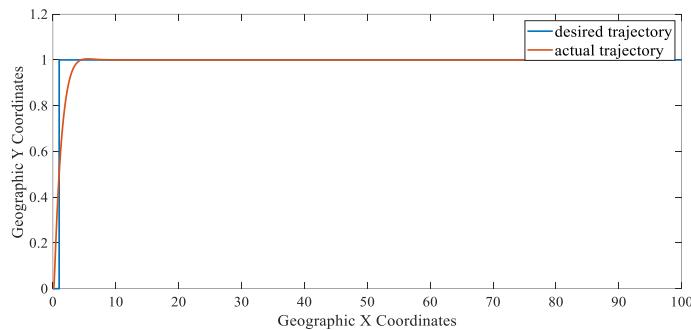


Figure 8. Linear path trajectory based on full states feedback linearization with observer based on LQR

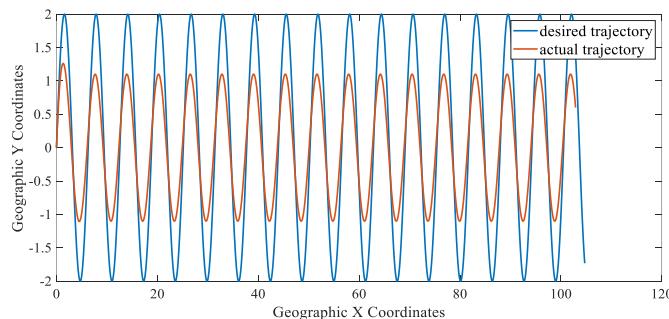


Figure 9. Sinusoidal path trajectory based on full states feedback linearization with observer LQR

The simulation result shows that full states feedback linearization with observer based on LQR improves performance and achieves stability for the linear trajectory as represent in Figure 8. Linear trajectory eliminates nonlinearities in the system and transform the system into a linear system. However, this approach cannot apply in a nonlinear system such as a nonlinear vehicle, because LQR is designed only for a linear system based on a quadratic cost function. So that, this approach unable to reach stability for nonlinear system as shown in Figures 9 and 10.

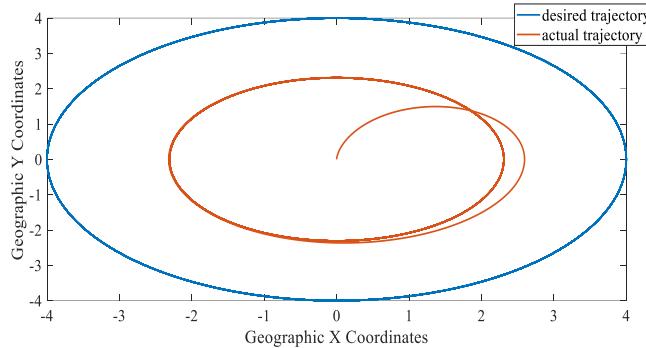


Figure 10. Circular path trajectory based on full states feedback linearization with observer LQR

The comparison Table 1 shows the comparison of the J cost function of both methods, state feedback linearization with observer and LQR. The comparison Table 1 shows that the state feedback linearization approach gives better results for a nonlinear system such as an autonomous vehicle due to its smaller J cost value compared with J cost function of the LQR. So, for a nonlinear system, a state feedback linearization is a better option than LQR. However, for a linear system, LQR gives better performance and optimization.

Table 1. Comparison of J cost function from two proposed methods

Input signal	J cost values	
	State feedback linearization	State feedback linearization based on LQR
Sinusoidal	4.5002	8.6541
circle	8.1415	8.8673

#### 4. CONCLUSION

This research proposed a trajectory tracking control for autonomous vehicles to track a curvature trajectory. The state feedback linearization and observer feedback are used to control the dynamic motion of the autonomous vehicle. The developed controller can control the model to navigate the desired trajectory. The simulation results show good vehicle tracking control performance and minimal lateral error. The LQR method cannot suit nonlinear systems such as vehicles due to its only design specific for linear systems.

The state feedback linearization approach and observer feedback can measure many of the states to predict the uncertain parameters from the model. The technique generates a stable steering angle with minimum error. In the future, vehicle performance will be improved, particularly for driving in a complex path scenario. So, MPC can be the solution for such as problem. Combining this approach with a convolutional neural network will give better results for autonomous vehicle technology's control and computer vision.

#### ACKNOWLEDGEMENT

We want to acknowledge the generous support received for this research project through the grant from The Minister of Education, Culture, Research, and Technology for funding research and community service in 2022 for the doctoral dissertation research skim. Financial support has played a crucial role in enabling the completion of this scientific paper and has significantly contributed to the advancement of knowledge in this field.

#### REFERENCES

- [1] X. Yang, L. Xiong, B. Leng, D. Zeng, and G. Zhuo, "Design, validation and comparison of path following controllers for autonomous vehicles," *Sensors (Switzerland)*, vol. 20, no. 21, pp. 1–24, 2020, doi: 10.3390/s20216052.
- [2] L. Xiong, X. Yang, X. Guirong, B. Leng, and X. Rhenxie, "Review on Motion Control of Autonomous Vehicles," *Journal of Mechanical Engineering*, vol. 56, no. 10, pp. 127–143, 2020, doi: 10.3901/JME.2020.10.127.
- [3] A. Mohammadzadeh and H. Taghavifar, "A robust fuzzy control approach for path-following control of autonomous vehicles," *Soft Computing*, vol. 24, no. 5, pp. 3223–3235, 2020, doi: 10.1007/s00500-019-04082-4.
- [4] R. Hussain and S. Zeadally, "Autonomous Cars: Research Results, Issues, and Future Challenges," in *IEEE Communications Surveys & Tutorials*, vol. 21, no. 2, pp. 1275–1313, 2019, doi: 10.1109/COMST.2018.2869360.
- [5] Q. Yao, Y. Tian, Q. Wang, and S. Wang, "Control Strategies on Path Tracking for Autonomous Vehicle: State of the Art and Future Challenges," *IEEE Access*, vol. 8, pp. 161211–161222, 2020, doi: 10.1109/ACCESS.2020.3020075.
- [6] M. Park and W. Han, "Development of Lateral Control System for Autonomous Vehicle Based on Adaptive Pure Pursuit

Algorithm," in *2014 14th International Conference on Control, Automation and Systems (ICCAS 2014)*, 2014, pp. 1443-1447, doi: 10.1109/ICCAS.2014.6987787.

[7] J. -B. Park, S. -H. Bae, B. -S. Koo, and J. -H. Kim, "When path tracking using look-ahead distance about the lateral error method and the velocity change method tracking comparison," *2014 14th International Conference on Control, Automation and Systems (ICCAS 2014)*, Gyeonggi-do, Korea (South), 2014, pp. 1643-1647, doi: 10.1109/ICCAS.2014.6987822.

[8] S. Allou and Y. Zennir, "A comparative study of PID-PSO and fuzzy controller for path tracking control of autonomous ground vehicles," in *ICINCO 2018-Proceedings of the 15th International Conference on Informatics in Control, Automation and Robotics*, 2018, vol. 1, pp. 296-304, Oct. 2018, doi: 10.5220/0006910902960304.

[9] S. Allou, Y. Zennir, and A. Belmeguenai, "Fuzzy logic controller for autonomous vehicle path tracking," in *2017 18th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA)*, Monastir, Tunisia, 2017, pp. 328-333, doi: 10.1109/STA.2017.8314969.

[10] C. Hu, R. Wang, F. Yan, and N. Chen, "Should the Desired Heading in Path Following of Autonomous Vehicles be the Tangent Direction of the Desired Path?," in *IEEE Transactions on Intelligent Transportation Systems*, vol. 16, no. 6, pp. 3084-3094, 2015, doi: 10.1109/TITS.2015.2435016.

[11] X. Zhang and X. Zhu, "Autonomous path tracking control of intelligent electric vehicles based on lane detection and optimal preview method," *Expert Systems with Applications*, vol. 121, pp. 38-48, 2019, doi: 10.1016/j.eswa.2018.12.005.

[12] C. Chatzikomis, A. Sorniotti, P. Gruber, M. Zanchetta, D. Willans, and B. Balcombe, "Comparison of Path Tracking and Torque-Vectoring Controllers for Autonomous Electric Vehicles," *IEEE Transactions on Intelligent Vehicles*, vol. 3, no. 4, pp. 559-570, 2018, doi: 10.1109/TIV.2018.2874529.

[13] F. Lin, L. Ni, Y. Zhao, and K. Wang, "Path Following Control of Intelligent Vehicles Considering Lateral Stability," *Journal of South China University of Technology*, vol. 46, no. 1, pp. 78-84, 2018.

[14] S. Xu and H. Peng, "Design, Analysis, and Experiments of Preview Path Tracking Control for Autonomous Vehicles," in *IEEE Transactions on Intelligent Transportation Systems*, vol. 21, no. 1, pp. 48-58, 2020, doi: 10.1109/TITS.2019.2892926.

[15] N. Wu, W. Huang, Z. Song, X. Wu, Q. Zhang, and S. Yao, "Adaptive dynamic preview control for autonomous vehicle trajectory following with DDP based path planner," in *IEEE Intelligent Vehicles Symposium, Proceedings*, Seoul, Korea (South), 2015, pp. 1012-1017, doi: 10.1109/IVS.2015.7225817.

[16] T. T. Tian, Z. S. Hou, S. D. Liu, and Z. D. Deng, "Model-free Adaptive Control Based Lateral Control of Self-driving Car," *Acta Automatica Sinica*, vol. 43, no. 11, pp. 1931-1940, 2017, doi: 10.16383/j.aas.2017.c160633.

[17] C. Sun, X. Zhang, L. Xi, and Y. Tian, "Design of a path-tracking steering controller for autonomous vehicles," *Energies*, vol. 11, no. 6, 2018, doi: 10.3390/en11061451.

[18] C. Dai, C. Zong, and G. Chen, "Path tracking control based on model predictive control with adaptive preview characteristics and speed-assisted constraint," *IEEE Access*, vol. 8, pp. 184697-184709, 2020, doi: 10.1109/ACCESS.2020.3029635.

[19] P. Tang, "Feedback linearization of mimo nonlinear system with measurable disturbance," in *Proceedings - 2020 12th International Conference on Measuring Technology and Mechatronics Automation, ICMTMA 2020*, 2020, vol. 8, pp. 744-749, doi: 10.1109/ICMTMA50254.2020.00162.

[20] A. Ziae, H. Kharrati, and M. Salim, "Feedback linearization based fault tolerant control for affine non-linear systems," in *2020 28th Iranian Conference on Electrical Engineering, ICEE 2020*, no. 1, pp. 1-4, 2020, doi: 10.1109/ICEE50131.2020.9260734.

[21] J. Umlauft, T. Beckers, M. Kimmel, and S. Hirche, "Feedback linearization using Gaussian processes," *2017 IEEE 56th Annual Conference on Decision and Control (CDC)*, Melbourne, VIC, Australia, 2017, pp. 5249-5255, doi: 10.1109/CDC.2017.8264435.

[22] M. Ahmad, A. Khan, M. A. Raza, and S. Ullah, "A study of state feedback controllers for pole placement," in *5th International Multi-Topic ICT Conference: Technologies for Future Generations, IMTIC 2018-Proceedings*, 2018, no. 1, doi: 10.1109/IMTIC.2018.8467276.

[23] J. Jiang and A. Astolfi, "Lateral Control of an Autonomous Vehicle," in *IEEE Transactions on Intelligent Vehicles*, vol. 3, no. 2, pp. 228-237, 2018, doi: 10.1109/TIV.2018.2804173.

[24] M. Rick, J. Clemens, L. Sommer, A. Folkers, K. Schill, and C. Büskens, "Autonomous Driving Based on Nonlinear Model Predictive Control and Multi-Sensor Fusion," *IFAC-PapersOnLine*, vol. 52, no. 8, pp. 458-473, 2019, doi: 10.1016/j.ifacol.2019.08.068.

[25] R. Liu, M. Wei, N. Sang, and J. Wei, "Research on Curved Path Tracking Control for Four-Wheel Steering Vehicle considering Road Adhesion Coefficient," *Mathematical Problems in Engineering*, vol. 2020, 2020, doi: 10.1155/2020/3108589.

[26] Z. Shen and T. Tsuchiya, "State Drift and Gait Plan in Feedback Linearization Control of a Tilt Vehicle," *8th International Conference on Control, Modeling and Computing (CMC 2022)*, At: Vienna, Austria, pp. 1-17, 2022, doi: 10.5121/csit.2022.120501.

[27] M. Baur and L. Bascetta, "An experimentally validated lqr approach to autonomous drifting stabilization," in *2019 18th European Control Conference, ECC 2019*, 2019, pp. 732-737, doi: 10.23919/ECC.2019.8795883.

[28] E. Alimohammadi, E. Khanmirza, and H. D. Gohari, "Velocity tracking of cruise control system by using feedback linearization method," *Automotive Science and Engineering*, vol. 8, no. 4, pp. 2826-2832, 2018.

[29] A. Patnaik *et al.*, "Design and Implementation of Path Trackers for Ackermann Drive based Vehicles," *arXiv*, 2020, doi: 10.48550/arXiv.2012.02978.

[30] A. Trotta, A. Cirillo, and M. Giorelli, "A feedback linearization based approach for fully autonomous adaptive cruise control," in *2019 18th European Control Conference, ECC 2019*, 2019, pp. 2614-2619, doi: 10.23919/ECC.2019.8795832.

[31] J. Cao, C. Song, S. Peng, S. Song, X. Zhang, and F. Xiao, "Trajectory Tracking Control Algorithm for Autonomous Vehicle Considering Cornering Characteristics," *IEEE Access*, vol. 8, pp. 59470-59484, 2020, doi: 10.1109/ACCESS.2020.2982963.

[32] R. C. Dorf and R. H. Bishop, "The Design of State Variable Feedback System," in *Modern Control System*, M. Horton, 13th ed. Hoboken, NJ, USA: Pearson, 2017, pp 784-852.

[33] M. Rokonuzzaman, N. Mohajer, S. Nahavandi, and S. Mohamed, "Review and performance evaluation of path tracking controllers of autonomous vehicles," *IET Intelligent Transport Systems*, vol. 15, no. 5, pp. 646-670, 2021, doi: 10.1049/itr2.12051.

[34] S. Bacha, M. Y. Ayad, R. Saadi, O. Kraa, A. Aboubou, and M. Y. Hammoudi, "Autonomous Vehicle Path Tracking Using Nonlinear Steering Control and Input-Output State Feedback Linearization," in *Proceedings of 2018 3rd International Conference on Electrical Sciences and Technologies in Maghreb, CISTEM 2018*, 2019, pp. 1-6, doi: 10.1109/CISTEM.2018.8613365.

[35] B. Sapiro, "LQR A State Space Optimal Control," *slidesplayer*, 2018. [Online]. Available: <https://slideplayer.com/slide/13131205/>, date accessed Dec. 20, 2021.

## BIOGRAPHIES OF AUTHORS



**Hasnawiya Hasan**     She received her bachelor's degree in Electrical Engineering from Hasanuddin University, Indonesia, in 2002, in the area of Electronics, Computer, and Control. Obtained, her Master's degree in Systems and Control from the University of New South Wales, Australia, with the scholarship awards from Australian Award Scholarship, her final project title is "Nonlinear Control of Bicycle", in 2005. She currently studies as a doctoral student in Electrical Engineering at Hasanuddin University, Indonesia, with a dissertation topic is Control of Autonomous Vehicle. She also currently works as a senior lecturer in Faculty of Engineering, at Hasanuddin University, South Sulawesi, Indonesia. She can be contacted at email: hasnahasan@unhas.ac.id.



**Faizal Arya Samman**     is a professor in the Department of Electrical Engineering, at Hasanuddin University, Indonesia. He received a bachelor of engineering degree in electrical engineering from Universitas Gadjah Mada, Indonesia, in 1999 and a master of engineering degree from Institut Teknologi Bandung with a Scholarship Award from the Indonesian Ministry of National Education, in 2002. He received his Ph.D. degree from Technische Universität Darmstadt, Germany with a scholarship award from Deutscher Akademischer Austausch-Dienst (DAAD, German Academic Exchange Service), in 2010. He worked toward the postdoctoral research in LOEWE-Zentrum AdRIA (Adaptronik-Research, Innovation, Application) within the research cooperation framework between Technische Universität Darmstadt and Fraunhofer Institut LBF in Darmstadt, in 2012. His research interests include integrated electronics, system on chip, computer system architecture, power electronics for renewable energy systems and electric vehicles, control systems, embedded systems based on microcontrollers and Field Programmable Gate Array (FPGA). He has been an invited professor at Frankfurt University of Applied Sciences conducting research in the field of high performance computer architecture for multiprocessor systems-on-chip. From 2019 until 2022, he has been assigned to be the DAAD Research Ambassador developing research networking and promoting Germany's Higher Educations and Research Institutes. He can be contacted at email: faizalas@unhas.ac.id.



**Muh Anshar**     he is an associate professor at the Department of Electrical, Faculty of Engineering Universitas Hasanuddin UNHAS Makassar Indonesia and also an Associate researcher at the Faculty of Engineering and Information Technology University of Technology Sydney UTS, Australia. Achieved a bachelor's degree, ST at the University of Hasanuddin, Indonesia, in the area of Control Systems and Design, in 2003. Obtained a master's degree in Computer Science, M.Sc. CS. by Research at the University of Technology Sydney UTS Australia in 2009, awarded an Australian Partnership Scholarship from the AUSAID, with the title of thesis A Directed Learning-based Evolutionary Approach for Legged Robot Motion. A Ph.D. in Computer Systems the University of Technology Sydney UTS Australia in the area of robotics, in 2017, awarded by the AUSAID in the scheme of the Australian Leadership Award with the project Evolving Robot Empathy through the Generation of Artificial Pain in an Adaptive Self-awareness Framework for Human-Robot Collaborative Tasks. Active research in the area of artificial intelligence, robotics, and recently in cognitive and social robotics with the particulars of the development of self-awareness concept into robot framework. Besides, several projects involving hardware robot development, embedded systems, and kinematics for robot gait development, with software development utilizing C, C++, R Programming, and Python. He is the head of the Indonesia-Australian Cognitive and Social Robotics Collaboration, which is a joint research work between the Faculty of Engineering UNHAS and the Faculty of Information and Technology UTS Australia which was formerly a joint research project in Advance Artificial Intelligence Research Group established in 2009 in the area of robotics. He can be contacted at email: anshar@unhas.ac.id.



**Rhiza S. Sadjad**     he is a retired senior lecturer in the Department of Electrical Engineering, at Hasanuddin University, South Sulawesi, Indonesia. He received his bachelor's degree in Electrical Engineering from the Institute of Technology Bandung, West Java, Indonesia, in 1981. His master's degree from the University of Wisconsin-Madison, USA, in 1989 and also Ph.D. degree in Electrical Engineering from University of Wisconsin-Madison, USA, in 1994. His research interest is in the field of automatic control and systems control. He can be contacted at email: rhiza@unhas.ac.id.