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Theoretical and experimental implementation of PID and sliding mode control on an inverted pendulum system

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ABSTRACT

This paper explores the theoretical and experimental implementation of proportional-integral-derivative (PID) and sliding mode control (SMC) on an inverted pendulum system, a well-known problem in control engineering that is inherently unstable and highly nonlinear. The primary objective of this study is to evaluate and compare the effectiveness of these two control strategies in achieving system stabilization and robustness against disturbances. The PID controller, widely utilized due to its straightforward design and implementation, is developed based on the linearized model of the inverted pendulum. On the other hand, the SMC technique, known for its robustness to system uncertainties and external disturbances, is employed to tackle the nonlinear nature of the system. The controllers are tested in both simulation and real-time experimental environments to ensure the reliability of the findings. The results from the experiments indicate that while the PID controller performs adequately under nominal conditions, it struggles to maintain stability when faced with parameter variations and external perturbations. In contrast, the SMC exhibits superior performance by consistently stabilizing the pendulum even under adverse conditions, demonstrating its robustness and effectiveness in managing nonlinear systems. This comparative analysis provides valuable insights into the practical applications of PID and SMC, highlighting the trade-offs between simplicity and robustness in control system design.

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1. INTRODUCTION

The control of inherently unstable systems poses a significant challenge in engineering and robotics, demanding sophisticated strategies to maintain stability and achieve desired performance. In this context, the inverted pendulum system stands as a quintessential example, characterized by its inherent instability and sensitivity to external disturbances. This research endeavours to address this challenge by exploring the theoretical and experimental implementation of two prominent control methodologies proportional-integral-derivative (PID) and sliding mode control (SMC) on an inverted pendulum [1].

The inverted pendulum, with its dynamic and unpredictable behaviour, serves as an ideal testbed for assessing the efficacy of control strategies in real-world scenarios. Theoretical analyses and simulations play a crucial role in the initial stages of this research, providing a comprehensive understanding of the mathematical model governing the inverted pendulum's dynamics. The application of PID control is meticulously studied, with an emphasis on parameter tuning to achieve optimal stability and performance metrics [2], [3].

Reference tracking, another critical aspect, involves the ability of a control system to accurately follow a specified reference signal or setpoint. Control architectures are engineered to achieve precise and reliable tracking, ensuring that the system responds appropriately to changes in the desired output. Additionally, robustness is a key criterion for evaluating control architectures. A robust control system should exhibit resilience to variations in system parameters and external factors, thereby ensuring consistent performance over a range of conditions. Two prominent control architectures, SMC and PID control, are often employed to address diverse control challenges. SMC is particularly suitable for systems characterized by uncertainties and disturbances. It excels in scenarios where precise control is essential despite the presence of unpredictable factors. On the other hand, PID control is widely embraced across industries owing to its simplicity and versatility. It is especially effective in systems with well-defined dynamics and where stringent control requirements may not be paramount [4], [5].

To bridge the gap between theory and practical application, real-time experiments are conducted on a physical inverted pendulum setup. This experimental phase involves system identification, data acquisition, and parameter adjustment for both PID and SMC. The outcomes of these experiments not only validate the theoretical findings but also provide valuable insights into the real-world challenges and nuances associated with implementing these control strategies.

The choice between SMC and PID control hinges on the specific characteristics of the system, the level of uncertainties involved, and the desired performance criteria. SMC is adept at handling uncertainties and disturbances in nonlinear systems, making it a preferred choice in situations where precise and robust control is imperative. In contrast, PID control is chosen for its simplicity and adaptability, making it suitable for a broad spectrum of applications [6], [7].

Recognizing the strengths of each method, there are instances where a combined approach, integrating elements of both sliding mode and PID control, is employed. This hybrid strategy aims to harness the advantages of both approaches, potentially offering improved performance and robustness. The integration of these control elements is tailored based on a comprehensive analysis of the system's dynamics and control requirements. The selection between SMC and PID control is contingent on the unique characteristics of the system, the precision level required, and the presence of uncertainties or disturbances. Each control architecture has distinct strengths, and the decision-making process involves a meticulous evaluation of the system's dynamics and control specifications. As control systems continue to play a pivotal role in shaping the performance of dynamic systems, the synergy between sliding mode and PID control emerges as a compelling avenue for achieving enhanced control outcomes in a diverse range of applications [8]–[10].

2. SIMULATION AND PID CONTROL IMPLEMENTATION OF INVERTED PENDULUM

The mathematical modelling of an inverted pendulum is essential for understanding its dynamic behaviour and designing control strategies to stabilize the system. The inverted pendulum consists of a rigid rod with a point mass at one end, which can move in a vertical plane. The mathematical model typically considers the following parameters: A rigid rod or stick with a mass at one end that is fastened to a pivot point makes up the inverted pendulum in Figure 1 despite the intrinsic instability brought on by gravity, the objective is to keep the pendulum in an upright position. The dynamics of the system can be described equations:

Cart variables:
x: horizontal position of the cart.
$\dot{x}$: velocity of the cart.

Pendulum variables:
θ: angular displacement of the pendulum from the vertical position.
$\dot{\theta}$: angular velocity of the pendulum.

Physical parameters:
m_c: mass of the cart.
m_p: mass of the point mass at the end of the pendulum.
l : length of the pendulum.
g : acceleration due to gravity.

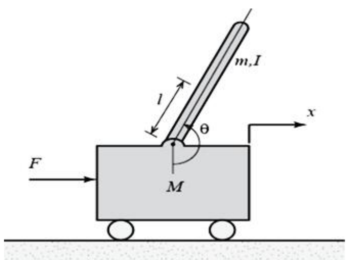

Figure 1. Freebody diagram of inverted pendulum with cart

The dynamic equations of motion for the inverted pendulum can be derived using Newton's second law and the principles of rotational motion. Assuming no friction or air resistance, the equations are typically represented in state-space form:

$$\dot{x} = \dot{x}$$
$$\ddot{x} = \frac{f + m_p sin\,(\theta)(l\dot{\theta}^2 + g cos\,(\theta))}{m_c + m_p sin^2\,(\theta)}$$
$$\dot{\theta} = \dot{\theta}$$
$$\ddot{\theta} = \frac{-f cos\,(\theta) - (m_c + m_p) g sin\,(\theta) + m_p l \dot{\theta}^2 cos\,(\theta) sin\,(\theta)}{l(m_c + m_p sin^2\,(\theta))}$$

where, f represents the force applied to the cart, which is a crucial parameter when considering control strategies. The linearized form of these equations, around an unstable equilibrium point, is often used for control design:

$$\dot{x} = Ax + Bu$$
$$\dot{x} = Cx + Du$$

where x is the state vector, u is the control input, and *A*, *B*, *C*, and *D* are matrices derived from the linearized equations. Understanding and utilizing these mathematical models are foundational for developing effective control strategies, such as PID or SMC, for stabilizing the inverted pendulum system. As shown in Figures 2 and 3, the inverted pendulum is a classic control system benchmark, challenging due to its inherent instability. Simulating it in MATLAB/Simscape lets you explore control design principles and evaluate different control strategies. Here's a breakdown of the key steps:

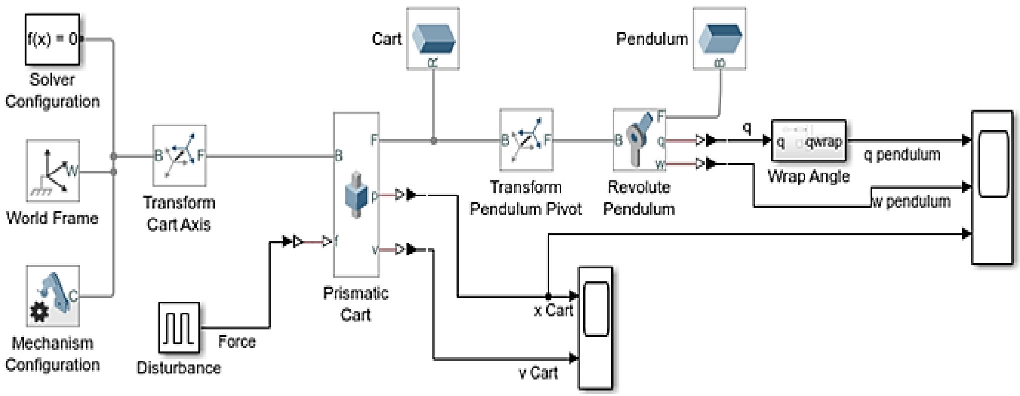

Figure 2. MATLAB/Simscape model of inverted pendulum with cart

2.1. Proportional-integral-derivative block simulink

PID controller dynamics: the dynamic behavior of a PID controller is governed by the weighted combination of these three terms, each adjusted by respective coefficients (Kp, Ki, and Kd) during the tuning process. The PID control output (u) is expressed equation:

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}$$

where, $e(t)$ denotes the error at time t, K_p, K_i, and K_d are the proportional, integral, and derivative gains, respectively.

- Tuning and optimization: the effectiveness of a PID controller lies in the careful tuning of its parameters to match the specific characteristics of the controlled system. Achieving an optimal balance between the proportional, integral, and derivative terms is essential for robust and stable performance. Numerous tuning methods, ranging from manual adjustment to advanced optimization algorithms, exist to fine-tune PID controllers based on the system's response characteristics, ensuring that the controller adapts to varying operating conditions.
- Applications and adaptability: PID controllers find widespread application across diverse industries due to their adaptability and simplicity. From temperature control in industrial processes to speed regulation

in motor drives, PID controllers offer a versatile solution for a myriad of control challenges. The architecture's straightforward structure and ease of implementation make it a popular choice in both academic settings and industrial environments [11].

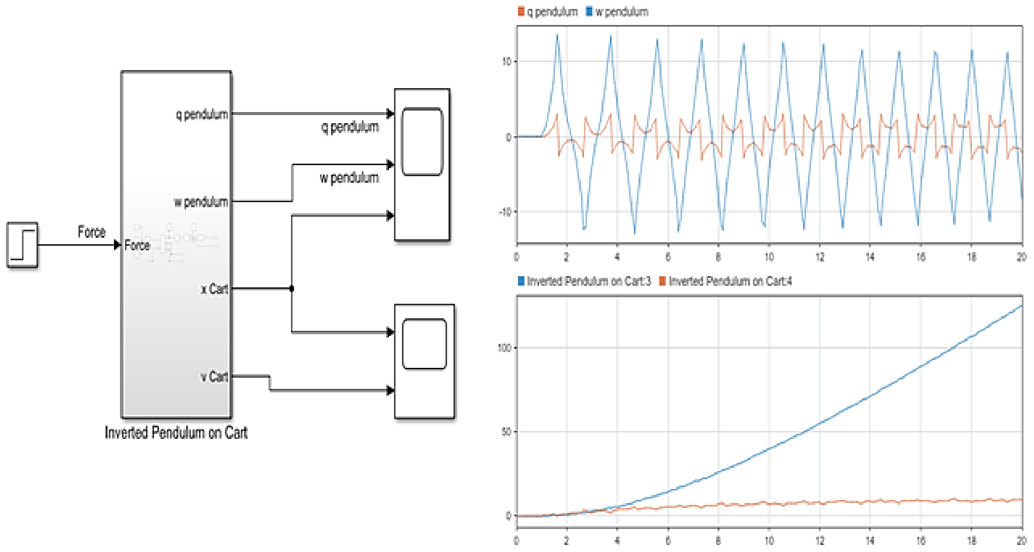

Figure 3. Simulation of inverted pendulum with cart

For the simulink, PID tuner offers a single-loop PID tuning technique that is quick and broadly applicable. The weighted sum of the input signal's derivative, integral, and input signal is the block output. The weights consist of the derivative, integral, and proportional gain parameters. The PID controller coefficients can be changed automatically or manually. Automatic tuning requires software known as simulink control design. See the select tuning technique parameter for further details on automatic tuning [12], [13]. Figures 4 and 5 shows result of PID implementation using MATLAB simulink toolbox. This shows a better performance of PID control (PID tuner block) (Kp=55.51, Kd=4.21, and Ki=114.48) compared to previously PID parameters (PID parameters Kp=100, Kd=5, and Ki=5).

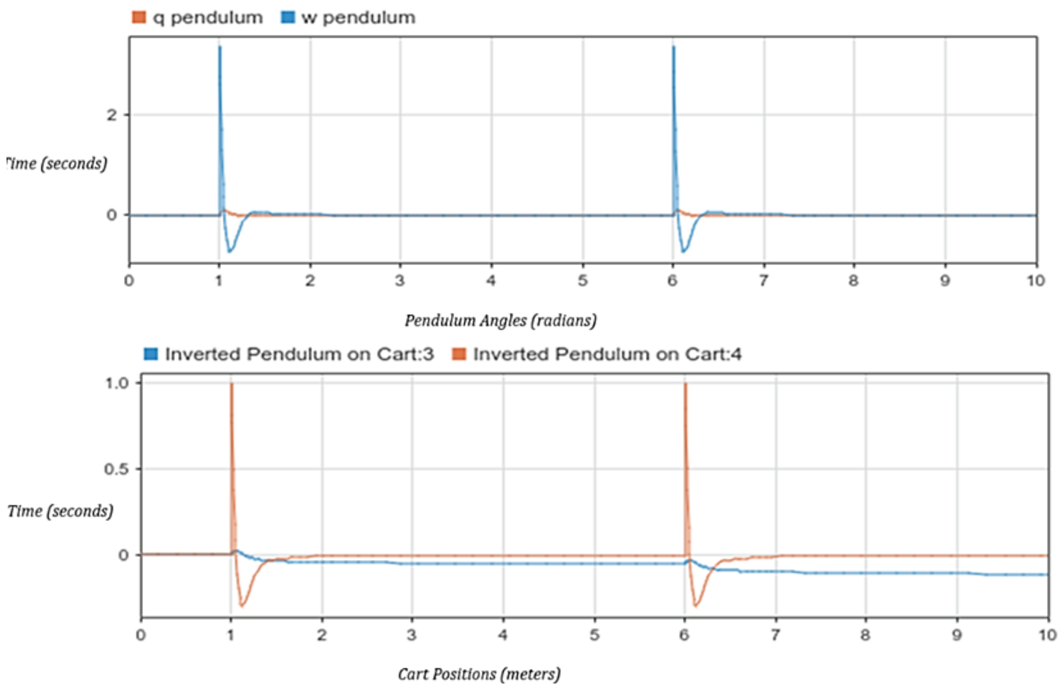

Figure 4. PID implementation on inverted pendulum (PID tuner block) (Kp=55.51, Kd=4.21, and Ki=114.48)

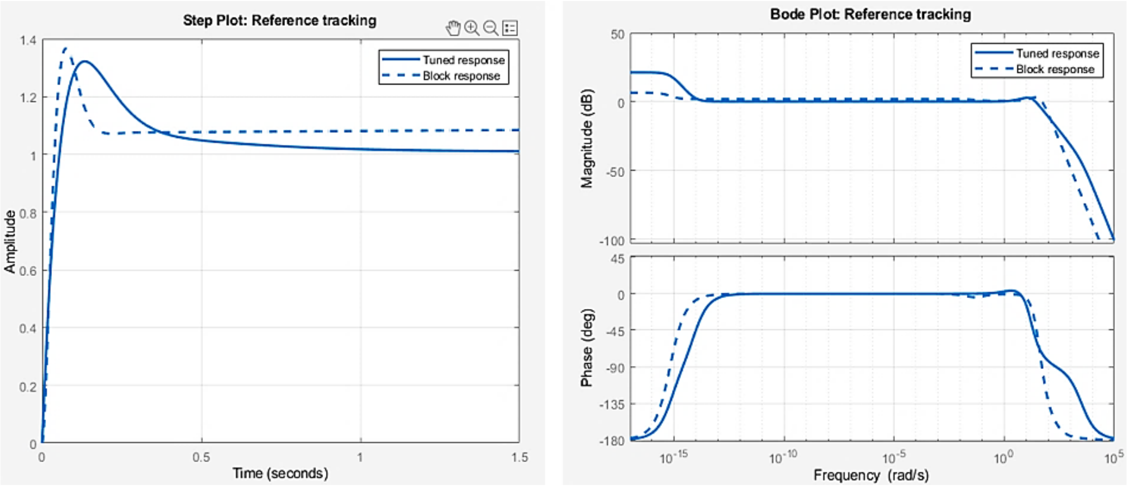

Figure 5. Step response to PID tunned plant and open loop response

3. SLIDING MODE CONTROL IMPLEMENTATION

SMC represents a sophisticated and technically advanced approach to control system design, renowned for its ability to handle complex nonlinearities and uncertainties in dynamic systems. At its core, SMC leverages the concept of a sliding surface, which serves as a dynamic equilibrium that the system trajectories aim to reach and maintain. This technical note delves into the intricacies of SMC, exploring its mathematical foundation, unique features, and practical applications. Mathematical foundation: the foundation of SMC lies in its mathematical representation of the sliding surface and the associated control law. Consider a system described by the state-space equations [14], [15].

3.1. Mathematical foundation

The foundation of SMC lies in its mathematical representation of the sliding surface and the associated control law. Consider a system described by the state-space equations:

$$\dot{x} = f(x, u)$$

where, x represents the state vector, and u is the control input.
The sliding surface, denoted as s, is typically defined as:

$$s = s(x) = h(x)$$

The control input u in SMC is designed to ensure that the system trajectories converge to and remain on the sliding surface. The control law is expressed as:

$$u = u(x) = -k\text{sgn}\,(s)$$

In this equation, k is a positive constant, and sgn $(\cdot)$ denotes the signum function. The negative sign ensures that the system follows the sliding surface.

Now, let's explore how SMC can be applied to stabilize the inverted pendulum. SMC involves creating a sliding surface, a hyperplane in the state space, such that the system trajectories are constrained to remain on this surface. The sliding surface is designed to drive the system towards a desired equilibrium point. In the case of the inverted pendulum, the objective is to keep it upright. The sliding surface for the inverted pendulum can be defined as:

$$s = \dot{\theta} + \lambda(\theta - \theta_{desired})$$

where, s is the sliding surface, λ is a positive constant, $\dot{\theta}$ is the angular velocity, and θ_{desired} is the desired angular position. The SMC law is then applied to ensure that the system trajectories converge to the sliding surface and remain on it. The control input u can be formulated as:

$$u = -\frac{ml^2}{u}\left(\frac{g}{l} sin\,(\theta) - ksgn\,(s)\right)$$

In this equation, k is a positive constant, and $\text{sgn}\ (s)$ is the signum function. The control input is designed to force the system trajectories to reach and stay on the sliding surface, effectively stabilizing the inverted pendulum.

3.1.1. Robustness to uncertainties

One of the distinctive features of SMC is its robustness to uncertainties and disturbances. The signum function introduces a robust control action that is insensitive to system uncertainties. The control law's ability to force the system onto the sliding surface regardless of uncertainties makes SMC particularly well-suited for applications where precise system modeling is challenging or impractical [16].

3.1.2. Dynamic behavior and precision

The dynamic behavior of SMC is governed by the sliding surface's design and the control law. The sliding motion induced by the control law ensures rapid and precise responses. The choice of the sliding surface function and the tuning of the control parameters, such as k, allow for tailoring the system's dynamic response to specific requirements. This dynamic precision makes SMC invaluable in applications where quick and accurate control is essential. The beauty of SMC lies in its ability to handle uncertainties and disturbances inherent in real-world systems. In the case of the inverted pendulum, external disturbances or modeling inaccuracies can be effectively countered by the SMC law, ensuring robust performance.

The inverted pendulum, a classic problem in control theory, poses inherent challenges due to its unstable nature. SMC, with its robust characteristics and ability to handle uncertainties, presents an effective solution for stabilizing and controlling the inverted pendulum. The application of SMC involves defining a sliding surface and formulating a control law to drive the system trajectories onto this surface. The equations provided offer insights into the dynamics and control strategy for the inverted pendulum under the influence of SMC [17], [18].

3.2. Simulation of control architectures

Simulation of control architectures is a nuanced and crucial facet in the realm of control system engineering, providing engineers with a powerful toolset to design, analyze, and optimize intricate control strategies. This detailed examination encompasses various technical dimensions, starting with the fundamental task of mathematical modeling. Mathematical models serve as the bedrock, encapsulating the intricacies of real-world systems through differential equations, state-space formulations, or transfer functions. For instance, in linear systems, transfer functions like $G(s) = \frac{Y(s)}{U(s)}$ act as bridges between input and output variables, allowing for a comprehensive representation.

Moving into the heart of the process, the modeling of the entire control system architecture is the next crucial step. This comprehensive model spans the entire spectrum, from the plant and sensors to actuators and the controller itself. In cases involving PID controllers, the transfer function $H(s)$, of the PID controller becomes an integral part of this model. The precision of this representation is paramount, as it lays the groundwork for subsequent simulation steps. Simulation tools and environments play a pivotal role, offering engineers platforms like MATLAB/Simulink, LabVIEW, and Modelica. MATLAB/Simulink, a prevalent choice, provides a graphical interface for constructing block diagrams that vividly represent control architectures. The visual nature of these tools simplifies the design and analysis of complex systems, allowing engineers to focus on the intricacies of the control strategy rather than grappling with complex coding [19], [20].

Dynamic simulation, a cornerstone of this process, involves an in-depth analysis of a system's behavior over time and in the frequency domain. Time domain analysis scrutinizes parameters such as overshoot, settling time, and transient response, providing insights into the system's dynamic characteristics. Meanwhile, frequency domain analysis, facilitated by tools like Bode plots and Nyquist diagrams, sheds light on how the system responds to different frequencies. These analyses collectively contribute to a holistic understanding of the control architecture's performance.

Validation, a critical step, involves comparing simulation models with experimental data obtained from physical prototypes. System identification techniques help adjust simulation parameters, ensuring a close alignment between virtual and real-world behavior. This iterative process enhances the accuracy of the simulation model, fostering confidence in its reliability [21], [22]. The convergence of virtual and real components is exemplified in advanced techniques like co-simulation and Hardware-in-the-Loop (HIL) testing. Co-simulation integrates multiple tools to model diverse aspects of a control system, while HIL testing introduces real hardware components into the simulation loop. This synergy ensures a more accurate representation of the control architecture's interaction with the physical system, particularly beneficial when a complete physical prototype is impractical [23]. Venturing into advanced simulation techniques, engineers leverage methodologies like Monte Carlo simulations to assess the robustness of control architectures under varying conditions and uncertainties [24], [25].

4. ROTARY INVERTED PENDULUM BALANCING-EXPERIMENTAL SETUP

The setup procedure for the rotary inverted pendulum balancing system involves a detailed assembly process to ensure the precise integration of hardware components, creating a robust experimental platform. Start by selecting a suitable rod or stick to represent the inverted pendulum, ensuring its length, mass distribution, and moment of inertia align with the specified requirements. Attach the rod to a stable base, establishing a pivot point that allows free rotation.

Incorporate the specified motor or actuator into the system, taking note of its nominal input voltage of 12 V and a nominal speed of 600 rpm. Proper alignment of the motor is crucial for optimal performance in controlling the motion of the inverted pendulum. Integrate position sensors, such as the vertical arm encoder with a count resolution of 10000 counts/rev, to measure the angular position of the pendulum accurately.

The motor amplifier, configured as a pulse width modulation (PWM) type, plays a pivotal role in controlling the motion of the pendulum. Ensure that the amplifier can handle a peak current of 3 A and a continuous current of 2 A, and connect it to the motor. Additionally, power the motor with the specified +12 V output from the amplifier. For future upgrades and enhanced data acquisition, include an onboard 1-channel 12-bit analog to digital converter (ADC) in the setup. Establish a robust ethernet LAN connection to facilitate high-speed communication between components. Assemble the entire system on the provided target board, which runs a Real-Time Application Linux (RTAI) with a target daemon for efficient communication. Make certain that the setup is fully compatible with the 20 Sim and 4C toolchain for seamless integration.

4.1. Data acquisition and real-time control

Data acquisition is a critical phase in the experimental process, requiring careful consideration of the high-performance processor and software interface. The ARM9 processor, equipped with a math co-processor, ensures real-time execution and precision in data acquisition. Utilize the software interface to monitor and log every variable available on the target system, providing a comprehensive overview of the system's behavior.

The preferred data file format is comma-separated values (.csv), allowing for easy accessibility and compatibility with various data analysis tools. Leverage the capabilities of the onboard 12-bit ADC to capture analog signals with high resolution, ensuring accurate representation of system dynamics. The ethernet LAN connection enhances data transfer speeds, facilitating the acquisition of large datasets with minimal latency. During the data acquisition phase, focus on monitoring key variables such as the rotary pendulum link mass, link length, motor parameters (nominal input voltage, speed, amplifier specifications), and encoder counts. Visualize these variables as waveforms for a detailed understanding of their behavior over time. Ensure that the control system model supplied with the board operates stably, providing a benchmark for subsequent experimentation.

4.2. Experimentation and analysis

The experimentation and analysis phase involves the implementation of control algorithms on the rotary inverted pendulum balancing system, leveraging the capabilities of the high-performance ARM9 processor. Utilize the supplied control system model to conduct experiments that assess the system's stability and response under varying conditions. The software interface plays a crucial role in this phase, allowing for the compilation of C code generated from the 20-sim control model and seamless downloading onto the board. This integration ensures a direct connection between the simulated model and the physical setup, enabling precise control and analysis.

Experiment with various control algorithms such as PID or SMC, exploiting the real-time capabilities of the ARM9 processor. Analyze the waveforms and logged data to evaluate the system's performance, making adjustments to control parameters as necessary. Considerhe impact of varying parameters, such as motor input voltage, link length, and control gains, on the system's behavior. The ethernet LAN connection facilitates high-speed communication, enabling quick response to dynamic changes during experimentation. Systematically vary parameters and conduct in-depth analysis to refine control strategies and optimize overall system performance. Through this detailed experimentation and analysis process, gain a comprehensive understanding of the intricate dynamics of the rotary inverted pendulum balancing system. Figure 6 and 7 shows experimental setup and blockdiagram of inverted pendulum setup.

The development and implementation of a control system for rotary inverted pendulum balancing represent a challenging yet intriguing endeavor in the realm of control theory and robotics. This section provides a detailed exploration of the specifications outlined for the complete system, offering insights into the crucial components and functionalities designed to demonstrate and exercise control system design for the rotary inverted pendulum. Figure 8 shows the block diagram representation of real-time PID control architecture on experimental setup of inverted pendulum with cart system.

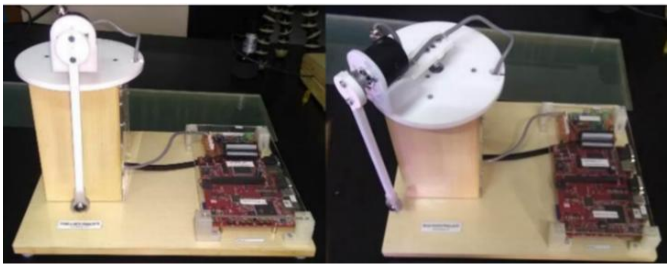

Figure 6. Experimental setup inverted pendulum

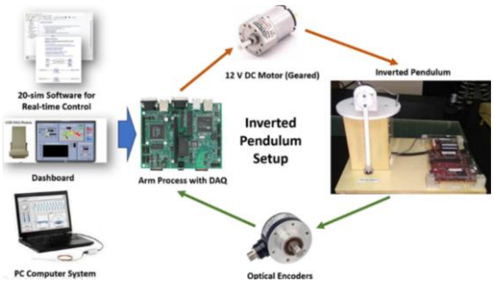

Figure 7. Block diagram for experimental setup

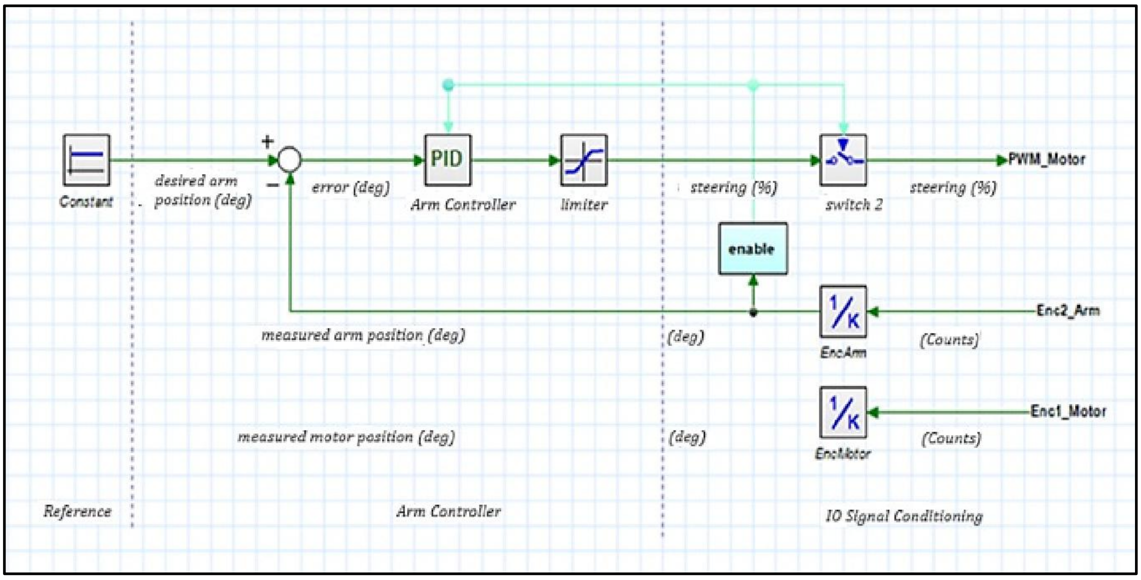

Figure 8. Real-time implementation of control architecture on inverted pendulum with cart system

5. RESULTS AND DISCUSSION

The theoretical analysis of the PID control implementation on the inverted pendulum system revealed that the proportional, integral, and derivative terms played distinct roles in achieving stability. The proportional term contributed to the quick response to deviations, the integral term assisted in eliminating steady-state errors, and the derivative term provided damping to prevent overshooting. The tuning parameters were optimized through simulations to ensure robust performance.

Figure 9 depicts the real-time performance of the PID control system applied to the inverted pendulum with a cart system. The results illustrate the effectiveness of the PID controller in stabilizing the pendulum. The proportional term of the PID controller enabled a rapid response to deviations, allowing the system to quickly correct any tilt of the pendulum. The integral term was crucial in addressing and eliminating steady-state errors, ensuring that the pendulum stayed in the desired upright position without persistent deviations. Additionally, the derivative term contributed to damping, minimizing overshooting and reducing settling time. The experimental results showed that the PID control system effectively managed response time, overshooting, and settling time, achieving performance close to theoretical predictions. Furthermore, the controller demonstrated its robustness by effectively compensating for external disturbances and uncertainties present in the experimental setup. This validation confirms the PID controller's capability to stabilize the inverted pendulum system under real-world conditions.

However, in the case of SMC, the theoretical analysis focused on designing a sliding surface to force the system onto a predefined trajectory. The SMC demonstrated inherent robustness against uncertainties and disturbances. The theoretical foundation emphasized the chattering phenomenon associated with SMC and strategies to mitigate its effects while maintaining the desired performance. Figure 10 presents the results of the SMC implementation for the inverted pendulum with a cart system. The sliding mode controller was designed to force the system onto a predefined trajectory, and the experimental results confirmed its effectiveness in achieving this goal. The controller displayed significant robustness against external disturbances and uncertainties, aligning with theoretical expectations. However, the chattering phenomenon, a known issue with SMC, was observed. Chattering refers to the rapid oscillations in the control signal caused by the switching nature of the SMC. While the sliding mode controller successfully guided the system to the desired trajectory, the chattering was noticeable and required mitigation strategies. Techniques such as smoothing the control signal or implementing boundary layers were employed to minimize the impact of chattering on system stability and performance. Overall, the experimental results validated the theoretical predictions of the SMC approach, demonstrating its robustness and effectiveness despite the challenges posed by chattering.

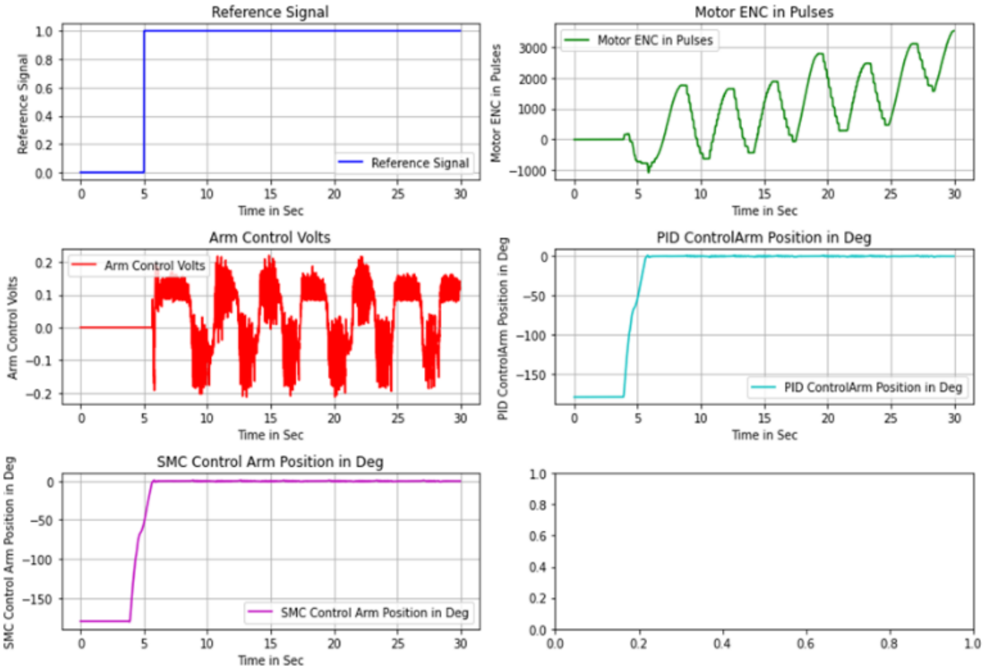

Figure 9. Real-time implementation of PID control architecture on inverted pendulum with cart system

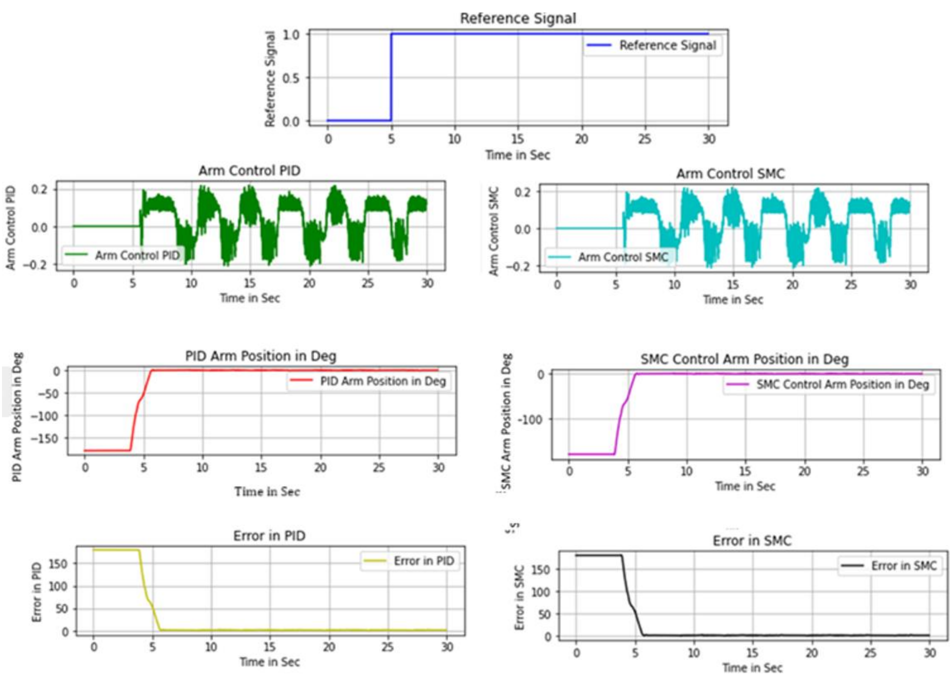

Figure 10. Real-time implementation of SMC architecture on inverted pendulum with cart system

6. CONCLUSION

The research addressed practical challenges encountered during the experimental implementation, including sensor noise, actuator limitations, and nonlinearities in the physical system. Strategies to mitigate these challenges were discussed, emphasizing the adaptability and limitations of each control strategy. The combined theoretical and experimental investigation provided valuable insights into the effectiveness of PID and SMC strategies for stabilizing an inverted pendulum system. The study contributes to the understanding of their respective strengths, weaknesses, and practical considerations, paving the way for informed choices in the control of similar dynamic systems. Further research avenues were suggested to enhance the performance and applicability of these control strategies in real-world scenarios. The study discussed the implications of the theoretical and experimental findings on the real-world applicability of PID and SMC for inverted pendulum systems. Considerations for industrial adoption and potential areas of improvement were explored.

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