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Taylor series linearization for fully fuzzy multi-objective fractional programming in educational systems

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ABSTRACT

This study examines the fully fuzzy multi-objective linear fractional programming problem (FFMOLFPP), whereby both the objective functions and restrictions incorporate fuzzy parameters represented as triangular fuzzy numbers (TFN), without converting them into crisp values. A hybrid solution approach is presented to tackle the intrinsic nonlinearity and uncertainty. Initially, the imprecise numbers are transformed into parametric representations via the γ-cut method. A first-order Taylor series expansion is subsequently utilized to linearize each fractional objective function around a fuzzy decision point. The linearized objectives are then consolidated by the weighted sum approach, transforming the multi-objective fuzzy model into a single-objective linear program. Numerical examples validate the strategy and demonstrate the improved accuracy and efficiency of the proposed methodology.

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1. INTRODUCTION

Linear fractional programming problems (LFPP), or ratio optimization, seek to optimize the ratio of two linear functions while complying with a specified set of constraints. LFPP is utilized a lot in healthcare, finance, industrial planning, and education. Combining fuzzy parameters makes it easier to cope with uncertainty, which is why LFPP is so valuable in real-world industrial and operational contexts. Many different ways have been found throughout the years to deal with different LFPP problems. The first research predominantly concentrated on distinctly defined problem contexts, with Charnes and Cooper [1] pioneering the innovative variable transformation approach that significantly impacted the evolution of multi-objective linear programming problems (MOLPP). Zadeh's [2] invention of fuzzy set theory signified a substantial advancement in the management of uncertainty, facilitating approaches for addressing multi-objective linear fractional programming problems (MOLFPP) inside fuzzy frameworks. In this context, Ganesan and Veeramani [3] tackled fuzzy linear programming problems (FLPP) employing symmetric triangular fuzzy numbers (TFN) without transforming them into crisp equivalents, thereby formulating fuzzy analogues of essential LPP theorems and facilitating direct solutions through primal and dual simplex methods.

Extensive research has focused on fuzzy and MOLFPP to improve decision-making under uncertainty. Veeramani and Sumathi [4] introduced a fuzzy mathematical programming technique to manage imprecise parameters without converting them into crisp values. Arya *et al.* [5] developed a fuzzy branch and bound method for MOLFPPs, enabling systematic enumeration of feasible regions under fuzzy data, while Pal *et al.* [6] incorporated imprecise aspiration levels through fuzzy goal programming. Perić *et al.* [7] proposed a fuzzy goal programming model with interval-valued parameters, effectively managing uncertainty without full defuzzification. Prasad *et al.* [8] applied fuzzy goal programming to achieve robust solutions under uncertain objectives and constraints. Bas and Ozkok [9] propose an iterative solution framework for fully fuzzy LFPP that solves successive crisp LPPs while retaining fuzziness, avoiding variable transformations typical in other methods. Namiq and Sadiq [10] introduce a fast solution technique for fuzzy-interval MOLFPP, using center-of-interval approximation to defuzzify parameters and reduce the problem to a crisp LP solved by the simplex method. Kumar and Mishra [11] develop a MOLFPP model using level-sets and chance-constrained programming to handle uncertainty and convert the problem into a crisp equivalent.

To manage fuzzy parameters in optimization, the γ-cut representation has been widely used. This method transforms fuzzy numbers into interval-valued data at different γ-confidence levels, as first structured by Puri *et al.* [12]. This approach allows for the systematic decomposition of fuzzy problems into deterministic subproblems. Karnik and Mendel [13] defined the centroid of a type-2 fuzzy set and provided methods to compute it using the KM algorithm. Foundational studies by Ma *et al.* [14] and Goetschel and Voxman [15] provided arithmetic and analytical tools to operate on fuzzy numbers using lattice and elementary fuzzy calculus, respectively. Dubois and Prade [16] extended this by introducing fuzzy integration over fuzzy intervals, a concept useful in the derivation of fuzzy parametric representations. Due to the nonlinear nature of fractional objectives, Taylor series expansion is employed to approximate the objectives around a feasible reference point. Toksari [17] was among the first to utilize first-order Taylor expansion for fuzzy fractional models. Earlier work by Güzel and Sivri [18] demonstrated the efficacy of Taylor series approximations in fuzzy MOLFPPs, thereby validating its practical use. This idea was extended by Pramanik *et al.* [19] and Kaur *et al.* [20] in transportation and multi objective fuzzy programming. Dangwal *et al.* [21] further confirmed that Taylor based linearization maintains computational tractability and precision in LFPP.

To handle FMOLFPPs, scalarization techniques like the weighted sum method have been widely adopted. Costa and Alves [22] highlighted its simplicity in capturing decision maker preferences, while Zare *et al.* [23] illustrated its use in interactive fuzzy decision-making systems. This method enables the conversion of fuzzy multi objective models into solvable single objective problems, which is particularly valuable when combined with γ cut and linearization techniques. The Charnes Cooper transformation has been extended to fuzzy environments to linearize fractional objectives under uncertainty. Yang *et al.* [24] demonstrated its effectiveness in solving fuzzy linear fractional problems. Loganathan and Ganesan [25] proposed an iterative method for solving FMOLFPP, where the fuzzy optimal solution of the reformulated problem represents the Pareto optimal solution of the original FMOLFPP. Maharana and Nayak [26] proposed a Taylor series expansion combined with a weighted sum approach to address FMOLFPP with TFN. Dalman [27] developed interactive fuzzy goal programming for bi-level fractional problems, introducing decentralized structures into fuzzy decision-making.Once the problem is solved at multiple γ levels, fuzzy reconstruction techniques are used to recover the original fuzzy solution. Govindhasamy and Kandasamy [28] developed an interior penalty function method for solving fuzzy nonlinear programming problems using TFN's and γ-cut analysis without defuzzification. Table 1 presents a summary of the literature on FMOLFPP, emphasizing the innovative aspects of recent methods relative to prior studies.

Despite these advancements, a persistent limitation remains- most FMOLFPP solution methods employ defuzzification at some stage, converting fuzzy parameters into crisp equivalents and thereby losing valuable uncertainty information. This diminishes the realism and interpretability of solutions in practical settings where fuzziness is inherent in the data. This study bridges the above gap by introducing a fuzziness-preserving framework for FMOLFPPs that:

- Retains the fuzziness of TFN through γ-parametric representation at all computational stages.
- Employs the Charnes–Cooper transformation to obtain the fuzzy decision point, combined with Taylor series approximation, without resorting to crisp conversion.
- Applies weighted sum scalarization to aggregate multiple objectives, solving the resulting parametric subproblems across several γ levels.

The suggested method gives fuzzy-optimal solutions that keep the uncertainty in mind, which makes FMOLFPP models more realistic and useful for making decisions.

Table 1. Research gap and contribution: enhancing fuzziness retention in FMOLFPP frameworks

Author(s)	Fuzzy	Approach	Without de-fuzzification
Pramanik *et al.* (2018) [19]	✓	Goal programming, Taylor's series	×
Yang *et al.* (2025) [24]	✓	Charnes–Cooper transformation	×
Dangwal *et al.* (2012) [21]	✓	Taylor linearization	×
Costa and Alves (2025) [22]	✓	Weighted sum, Taylor approximation	×
Arora and Gupta (2011) [30]	✓	Interactive fuzzy bi-level goal programming	×
Bas and Ozkok (2024) [9]	✓	Iterative framework preserving fuzziness	×
Kumar and Mishra (2025) [11]	✓	Fuzzy-stochastic linear fractional programming	×
Namiq and Sadiq (2025) [10]	✓	Fuzzy interval multi-objective FP	×
Proposed technique	✓	γ-parametric representation, Taylor linearization for fractional objectives, Weighted sum scalarization across several γ levels	✓

This paper is structured in the following way: section 2 gives a short summary of fuzzy numbers, focusing on how to rank them and perform parametric arithmetic operations with them. Section 3 lays out the basic structure of the FMOLFPP, discusses the important theorems, and explains the suggested method in depth. Section 4 illustrates the process with a numerical example, followed by a discourse on the results that highlight the approach's efficacy. Section 5 wraps up the study by summarizing the main results and suggesting some possible avenues for further research. Finally, section 6 presents the conclusion, highlighting the key findings and the overall contributions of the study.

2. PRELIMINARIES

a brief overview of essential definitions, notations, and foundational theories relevant to the study.

Definition 2.1. A fuzzy set $\tilde{\mathcal{U}}$ defined on the real number set $\mathbb{R}$ is characterized by a membership function $\mu_{\tilde{\mathcal{U}}} : \mathbb{R} \to [0, 1]$ that satisfies the following conditions [2]:

1) Normality: there exists $x_0 \in \mathbb{R}$ such that $\mu_{\tilde{\mathcal{U}}}(x_0) = 1$
2) Convexity: for all $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0, 1]$, the membership function satisfies: $\mu_{\tilde{\mathcal{U}}}(\lambda x_1 + (1 - \lambda)x_2) \geq \min\{\mu_{\tilde{\mathcal{U}}}(x_1), \mu_{\tilde{\mathcal{U}}}(x_2)\}$.
3) Upper semi-continuity: the function $\mu_{\tilde{\mathcal{U}}}(x)$ is upper semi-continuous, meaning that for any sequence $x_n \to x$, $\limsup_{n\to\infty} \mu_{\tilde{\mathcal{U}}}(x_n) \leq \mu_{\tilde{\mathcal{U}}}(x)$.
4) Bounded support: the support of $\tilde{\mathcal{U}}$, defined as supp$(\tilde{\mathcal{U}}) = \{x \in \mathbb{R} \mid \mu_{\tilde{\mathcal{U}}}(x) > 0\}$, is a bounded subset of $\mathbb{R}$.

The set of all fuzzy numbers on $\mathbb{R}$ is denoted by $\mathfrak{F}(\mathbb{R})$.

Definition 2.2. A TFN $\tilde{\mathcal{U}} \in \mathfrak{F}(\mathbb{R})$ is defined by the membership function and is denoted as $\tilde{\mathcal{U}} = (u_1, u_2, u_3)$ [28]:

$$\mu_{\tilde{\mathcal{U}}}(x) = \begin{cases} \dfrac{x - u_1}{u_2 - u_1}, & u_1 \leq x \leq u_2, \\ \dfrac{u_3 - x}{u_3 - u_2}, & u_2 \leq x \leq u_3, \\ 0, & \text{otherwise}, \end{cases}$$

Definition 2.3. A fuzzy number $\tilde{\mathcal{U}}$ can be characterized by a pair of functions $(\underline{u}(\gamma), \overline{u}(\gamma))$, defined over $\gamma \in [0, 1]$, which satisfy the following properties [28]:

- The lower bound function $\underline{u}(\gamma)$ is bounded, non-decreasing, and left-continuous.
- The upper bound function $\overline{u}(\gamma)$ is bounded, non-increasing, and left-continuous.
- For every $\gamma \in [0, 1]$, the inequality $\underline{u}(\gamma) \leq \overline{u}(\gamma)$ holds.

Definition 2.4. Let $\tilde{\mathcal{U}} = (u_1, u_2, u_3)$ be a TFN. Its γ-parametric form is [28]:

$$\underline{u}(\gamma) = u_1 + (u_2 - u_1)\gamma, \quad \overline{u}(\gamma) = u_3 - (u_3 - u_2)\gamma, \quad \gamma \in [0, 1].$$

This can also be expressed as $\tilde{\mathcal{U}} = (u_0,\ u_L,\ u_R)$,
where $u_L = u_2 - u_1, \quad u_R = u_3 - u_2, \quad u_0 = \frac{\underline{u}(1)+\overline{u}(1)}{2} = u_2$.

Definition 2.5. Ma *et al.* [14] proposed a parametric representation for fuzzy numbers, characterized by a location index and a fuzziness index function. The location indices are handled using classical arithmetic operations. The fuzziness indices follow lattice-based operations $\mathcal{L}$, i.e., for $u, v \in \mathcal{L}$. We define $u \vee v = \max\{u, v\}, \quad u \wedge v = \min\{u, v\}$ Let $\tilde{\mathcal{U}} = (u_0, u_*, u^*)$ and $\tilde{\mathcal{V}} = (v_0, v_*, v^*)$ be two fuzzy numbers in $\mathfrak{F}(\mathbb{R})$, and let $* \in \{+, -, \times, \div\}$. Then, the arithmetic operation between them is defined as $\tilde{\mathcal{U}} * \tilde{\mathcal{V}} = (u_0 * v_0, \max\{u_*, v_*\}, \max\{u^*, v^*\})$. In particular:

- Addition : $\tilde{\mathcal{S}} + \tilde{\mathcal{Q}} = (u_0 + v_0, \max\{u_*, v_*\}, \max\{u^*, v^*\})$
- Subtraction : $\tilde{\mathcal{S}} - \tilde{\mathcal{Q}} = (u_0 - v_0, \max\{u_*, v_*\}, \max\{u^*, v^*\})$
- Multiplication : $\tilde{\mathcal{S}} \times \tilde{\mathcal{Q}} = (u_0 \times v_0, \max\{u_*, v_*\}, \max\{u^*, v^*\})$
- Division : $\tilde{\mathcal{S}} \div \tilde{\mathcal{Q}} = \left(\frac{u_0}{v_0}, \max\{u_*, v_*\}, \max\{u^*, v^*\}\right)$, provided $v_0 \neq 0$.

Definition 2.6. Ranking fuzzy numbers plays a vital role in decision-making under fuzzy environments. This study adopts an efficient ranking method based on the graded mean approach [28]. Consider a TFN $\tilde{\mathcal{U}} = (u_0, u_*, u^*) \in \mathfrak{F}(\mathbb{R})$. The ranking function $R : \mathfrak{F}(\mathbb{R}) \to \mathbb{R}$ is given by:

$$R(\tilde{\mathcal{U}}) = \frac{u^* + 4u_0 + u^*}{6}.$$

For any two TFN's $\tilde{\mathcal{S}} = (u_0, u_*, u^*)$ and $\tilde{\mathcal{V}} = (v_0, v_*, v^*)$ in $\mathfrak{F}(\mathbb{R})$, their comparison is carried out as follows:

- If $R(\tilde{\mathcal{U}}) < R(\tilde{\mathcal{V}})$, then $\tilde{\mathcal{U}} \prec \tilde{\mathcal{V}}$,
- If $R(\tilde{\mathcal{U}}) > R(\tilde{\mathcal{V}})$, then $\tilde{\mathcal{U}} \succ \tilde{\mathcal{V}}$,
- If $R(\tilde{\mathcal{U}}) = R(\tilde{\mathcal{V}})$, then $\tilde{\mathcal{U}} \approx \tilde{\mathcal{V}}$.

Definition 2.7. A fuzzy-valued function $\tilde{f} : \mathbb{R} \to \mathfrak{f}(\mathbb{R})$ is defined to be continuous at $g_0 \in \mathbb{R}$ if, for every $\varepsilon > 0$, there exists a $\delta > 0$ such that $D(\tilde{f}(g), \tilde{f}(g_0)) < \varepsilon$ whenever $|g - g_0| < \delta$, where D denotes a distance metric on $\mathfrak{f}(\mathbb{R})$ [16], [28].

Definition 2.8. Let $\tilde{\mathcal{U}} = (\underline{u}(\gamma), \overline{u}(\gamma))$ and $\tilde{\mathcal{V}} = (\underline{v}(\gamma), \overline{v}(\gamma))$ be two fuzzy numbers. Then the distance between $\tilde{\mathcal{U}}$ and $\tilde{\mathcal{V}}$ is defined as $(\tilde{\mathcal{U}}, \tilde{\mathcal{V}}) = \sup_{0\le\gamma\le1} \{\max(|\underline{u}(\gamma) - \underline{v}(\gamma)|, |\overline{u}(\gamma) - \overline{v}(\gamma)|)\}$ [15], [28].

Definition 2.9. Let $\tilde{f} : \mathbb{R} \to \mathfrak{f}(\mathbb{R})$ be a fuzzy-valued function. The derivative of $\tilde{f}'$ at $g_0 \in \mathbb{R}$ is defined as $\tilde{f}'(g_0) = \lim_{h\to0} \frac{\tilde{f}(g_0+h)-\tilde{f}(g_0)}{h}$, provided the limit exists with respect to the distance metric D [15], [28], [29].

Theorem 2.10. Let $\tilde{f} : [u, v] \to \mathfrak{F}(\mathbb{R})$ be a fuzzy-valued function defined on a closed interval $[u, v]$, such that its derivative exists at a point $g \in [u, v]$. Then the following properties hold [15], [28]:

- The derivative of the location index function $\tilde{f}_0(g)$ exists, and it coincides with the location index of the derivative of $\tilde{f}$, i.e., $[\tilde{f}'(g)]_0 = \tilde{f}_0'(g)$.
- The fuzziness of the derivative from the left $(\tilde{f}'_-(g))_*$ and right $(\tilde{f}'_+(g))^*$ is bounded above by:
  $(\tilde{f}'_-(g))_* \le \inf_{h>0} \sup_{\gamma\in[-h,0]} \left((\tilde{f}(g + \gamma))_*\right), \quad and \quad (\tilde{f}'_+(g))^* \le \inf_{h>0} \sup_{\gamma\in[0,h]} \left((\tilde{f}(g + \gamma))^*\right)$

3. MATHEMATICAL MODEL

This section outlines the formal structure of the fully fuzzy multi-objective linear fractional programming problem (FFMOLFPP) by incorporating fuzzy parameters and multiple fractional objectives. It further develops the corresponding mathematical model and introduces the fully fuzzy Taylor series theorem, providing a solid basis for the proposed solution approach.

Theorem 3.1. First-order Taylor series approximation for fully fuzzy-valued multivariable functions

Let $\tilde{F} : \tilde{\mathbb{R}}^n \to \tilde{\mathbb{R}}$ be a fuzzy-valued function, i.e., a function that maps fuzzy vectors to fuzzy numbers. Suppose $\tilde{F}$ is Hukuhara (H-) differentiable and continuously fuzzy differentiable in a neighborhood of a fuzzy decision point $\tilde{x}^{(0)} = (\tilde{x}_1^{(0)}, \tilde{x}_2^{(0)}, \ldots, \tilde{x}_n^{(0)}) \in \tilde{\mathbb{R}}^n$. Then, for any point $\tilde{X} = (\tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n) \in \tilde{\mathbb{R}}^n$ close to $\tilde{x}^{(0)}$, the first-order fuzzy Taylor approximation of $\tilde{F}$ around $\tilde{x}^{(0)}$ is:

$$\tilde{F}(\tilde{X}) \approx \tilde{F}(\tilde{x}^{(0)}) \oplus \sum_{i=1}^{n} \frac{\partial \tilde{F}}{\partial \tilde{x}_i}(\tilde{x}^{(0)}) \otimes (\tilde{X}_i - \tilde{x}_i^{(0)}).$$

Expressed as:

$$\tilde{F}(\tilde{X}) \approx \tilde{F}(\tilde{x}^{(0)}) \oplus \nabla\tilde{F}(\tilde{x}^{(0)})^T \otimes (\tilde{X} - \tilde{x}^{(0)}),$$

where:

$$\nabla\tilde{F}(\tilde{x}^{(0)}) = \begin{bmatrix} \frac{\partial\tilde{F}}{\partial\tilde{x}_1}(\tilde{x}^{(0)}) \\ \frac{\partial\tilde{F}}{\partial\tilde{x}_2}(\tilde{x}^{(0)}) \\ \vdots \\ \frac{\partial\tilde{F}}{\partial\tilde{x}_n}(\tilde{x}^{(0)}) \end{bmatrix} \in \tilde{\mathbb{R}}^n \text{ is the fuzzy gradient vector,}$$

$$\tilde{X} - \tilde{x}^{(0)} = \begin{bmatrix} \tilde{X}_1 - \tilde{x}_1^{(0)} \\ \tilde{X}_2 - \tilde{x}_2^{(0)} \\ \vdots \\ \tilde{X}_n - \tilde{x}_n^{(0)} \end{bmatrix} \text{ is the fuzzy displacement vector,}$$

$\oplus$ denotes fuzzy addition, $\otimes$ denotes fuzzy–fuzzy multiplication defined via Hukuhara difference.

Proof: Let $\tilde{F} : \tilde{\mathbb{R}}^n \to \tilde{\mathbb{R}}$ be a fuzzy-valued function such that for each $\tilde{X} \in \tilde{\mathbb{R}}^n$, $\tilde{F}(\tilde{X})$ is a fuzzy number. Assume $\tilde{F}$ is H-differentiable at $\tilde{x}^{(0)}$ and the partial derivatives $\frac{\partial\tilde{F}}{\partial\tilde{x}_i}$ exist and are continuous in a neighborhood of $\tilde{x}^{(0)}$. Define the fuzzy increment

$$\tilde{h} = \tilde{X} - \tilde{x}^{(0)} = (\tilde{h}_1, \tilde{h}_2, \ldots, \tilde{h}_n), \quad \tilde{h}_i = \tilde{X}_i - \tilde{x}_i^{(0)}.$$

By the fuzzy Taylor expansion (Hukuhara sense), for sufficiently small $\tilde{h}$:

$$\tilde{F}(\tilde{x}^{(0)} + \tilde{h}) = \tilde{F}(\tilde{x}^{(0)}) \oplus \sum_{i=1}^{n} \frac{\partial\tilde{F}}{\partial\tilde{x}_i}(\tilde{x}^{(0)}) \otimes \tilde{h}_i \oplus \tilde{R}_1(\tilde{h}),$$

where the remainder term $\tilde{R}_1(\tilde{h})$ satisfies

$$\lim_{\|\tilde{h}\|\to 0} \frac{\|\tilde{R}_1(\tilde{h})\|}{\|\tilde{h}\|} = \tilde{0}, \quad \|\tilde{R}_1(\tilde{h})\| = \sup_{0\le\gamma\le 1} \max(|\underline{R}_1(\gamma)|, |\overline{R}_1(\gamma)|),$$

using the Hausdorff metric for fuzzy norms. Neglecting the remainder term for small $\tilde{h}$ gives the first-order approximation as:

$$\tilde{F}(\tilde{X}) \approx \tilde{F}(\tilde{x}^{(0)}) \oplus \sum_{i=1}^{n} \frac{\partial\tilde{F}}{\partial\tilde{x}_i}(\tilde{x}^{(0)}) \otimes (\tilde{X}_i - \tilde{x}_i^{(0)}).$$

Therefore:

$$\tilde{F}(\tilde{X}) \approx \tilde{F}(\tilde{x}^{(0)}) \oplus \nabla\tilde{F}(\tilde{x}^{(0)})^T \otimes (\tilde{X} - \tilde{x}^{(0)})$$

3.1. Proposed model

In this section, FFMOLFPP optimizes multiple fuzzy linear fractional objectives over a fuzzy feasible region by approximating each objective via a first-order Taylor series at each γ-cut and aggregating them into a single fuzzy problem, yielding fuzzy optimal solutions that capture inherent uncertainty.

3.1.1. Fully fuzzy multi-objective linear fractional programming

Consider the fully fuzzy multi-objective problem:

$$\tilde{Z}(\tilde{x}) = (\tilde{Z}_1(\tilde{x}), \tilde{Z}_2(\tilde{x}), \ldots, \tilde{Z}_k(\tilde{x})), \quad \tilde{x} \in \tilde{R} = \{\tilde{x} \in \tilde{\mathbb{R}}^n \mid \tilde{A} \otimes \tilde{x} \le \tilde{b},\ \tilde{x} \ge \tilde{0}\},$$

where each fuzzy objective function is expressed as a ratio of fuzzy linear functions:

$$\tilde{Z}_i(\tilde{x}) = \frac{\tilde{C}_i^T \otimes \tilde{x} + \tilde{\gamma}_i}{\tilde{D}_i^T \otimes \tilde{x} + \tilde{\beta}_i}, \quad i = 1, 2, \ldots, k,$$

with fuzzy coefficients $\tilde{C}_i, \tilde{D}_i \in \tilde{\mathbb{R}}^n$ and $\tilde{\gamma}_i, \tilde{\beta}_i \in \tilde{\mathbb{R}}$. It is assumed that the denominators remain strictly positive over the feasible fuzzy region $\tilde{R}$.

3.1.2. Determining the fuzzy decision point via Charnes–Cooper transformation

At each fixed $\gamma \in [0,1]$, the Charnes–Cooper transformation is applied to convert the fuzzy fractional objectives into an equivalent fuzzy linear form. This step provides a suitable fuzzy decision point $\tilde{x}_0^{(\gamma)}$ in the vicinity of the optimal region, which is then used for further approximation.

3.1.3. Handling nonlinearity and fuzziness via Taylor series

At each fixed $\gamma \in [0,1]$, express $\tilde{Z}_i(\tilde{x})$ in its parametric form using TFNs. For a suitable fuzzy decision point $\tilde{x}_0^{(\gamma)}$ near the optimal region, the first-order Taylor series approximation of $\tilde{Z}_i^{(\gamma)}(\tilde{x})$ is:

$$\tilde{Z}_i^{(\gamma)}(\tilde{x}) \approx T_i^{(\gamma)}(\tilde{x}) = \tilde{F}_i(\tilde{x}_0^{(\gamma)}) \oplus \nabla \tilde{F}_i(\tilde{x}_0^{(\gamma)})^T \otimes (\tilde{x} - \tilde{x}_0^{(\gamma)}).$$

Replacing each fuzzy fractional objective with its Taylor approximation leads to the approximated multi-objective problem:

$$\tilde{T}^{(\gamma)}(\tilde{x}) = (T_1^{(\gamma)}(\tilde{x}), T_2^{(\gamma)}(\tilde{x}), \ldots, T_k^{(\gamma)}(\tilde{x})), \quad \tilde{x} \in \tilde{R}_\gamma = \{\tilde{x} \in \tilde{\mathbb{R}}^n \mid A_\gamma \otimes \tilde{x} \leq b_\gamma,\ \tilde{x} \geq \tilde{0}\}.$$

3.1.4. Aggregation via weighted sum

For predefined weights $w_i \geq 0$ with $\sum_{i=1}^{k} w_i = 1$, the multi-objective fuzzy problem can be transformed into the single-objective fuzzy problem at each γ-level becomes:

$$Z^{(\gamma)}(\tilde{x}) = \sum_{i=1}^{k} w_i \, T_i^{(\gamma)}(\tilde{x}), \quad \tilde{x} \in \tilde{R}_\gamma.$$

Solving these deterministic problems across all γ-levels yields a set of fuzzy optimal solutions, effectively incorporating fuzziness into the optimization process.

3.2. Algorithm for solving FFMOLFPP using γ-parametric Taylor series linearization

The following algorithm summarizes the procedure for solving a FFMOLFPP using γ-parametric TFN, Taylor series linearization, and Charnes-Cooper transformation:

Step 1: formulate the FFMOLFPP with k fuzzy fractional objective functions and fuzzy constraints.
Step 2: express all TFN's in their γ-parametric forms.
Step 3: apply the Charnes–Cooper transformation to convert each fuzzy fractional objective into a linear form. Solve the resulting linear program to obtain a fuzzy feasible solution $x_0^{(\gamma)}$, which serves as the expansion point for Taylor series linearization.
Step 4: using the expansion point $\tilde{x}_0^{(\gamma)}$. Each fuzzy objective function is linearized using a first-order Taylor series expansion, yielding $T_i^{(\gamma)}(x)$.
Step 5: construct a single aggregated objective function by applying the weighted sum method to the linearized objective functions.
Step 6: first, solve the resulting single-objective linear program using the classical simplex method. Fuzziness is then incorporated by evaluating the solution across multiple γ-levels (e.g., $\gamma = 0,\ 0.25,\ 0.5,\ 0.75,\ 1$) and substituting them into the bounds of the fuzzy parameters. Reconstruct the fuzzy optimal solution set from the collection of solutions obtained at different γ-levels. The overall procedure of the proposed methodology is illustrated in the flowchart shown in Figure 1 (see in Appendix).

4. NUMERICAL ILLUSTRATION

To demonstrate the effectiveness of the proposed method, a numerical example is presented. It highlights how the FFMOLFPP approach can be applied in practice to obtain efficient solutions.

Example 1: to illustrate the usefulness of the suggested approach we consider a numerical example adapted from Arya *et al.* [5] and Güzel and Sivri [18].

$$\max\{\tilde{z_1}(\tilde{x}), \tilde{z_2}(\tilde{x})\} = \left\{ \begin{array}{c} \dfrac{-(2,3,4)\tilde{x_1} + (1,2,3)\tilde{x_2}}{(0.5,1,1.5)\tilde{x_1} + (0.5,1,1.5)\tilde{x_2} + (2,3,4)}, \\ \dfrac{(6,7,8)\tilde{x_1} + (0.5,1,1.5)\tilde{x_2}}{(4,5,6)\tilde{x_1} + (1,2,3)\tilde{x_2} + (0.5,1,1.5)} \end{array} \right\} \tag{1}$$

Subject to:

$$-(0.5,1,1.5)\tilde{x}_1+(0.5,1,1.5)\tilde{x}_2 \le -(0.5,1,1.5)$$
$$(1,2,3)\tilde{x}_1+(2,3,4)\tilde{x}_2 \le (14,15,16)$$
$$-(0.5,1,1.5)\tilde{x}_1 \le -(2,3,4)$$
$$\tilde{x}_1,\tilde{x}_2 \ge 0$$

Solution: the parametric form of the given FFMOLFPP is given by:

$$\max\{\tilde{z_1}(\tilde{x}),\tilde{z_2}(\tilde{x})\}=\left\{\begin{array}{c}\dfrac{-(3,1-\gamma,1-\gamma)\tilde{x_1}+(2,1-\gamma,1-\gamma)\tilde{x_2}}{(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{x_1}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{x_2}+(3,1-\gamma,1-\gamma)},\\ \dfrac{(7,1-\gamma,1-\gamma)\tilde{x_1}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{x_2}}{(5,1-\gamma,1-\gamma)\tilde{x_1}+(2,1-\gamma,1-\gamma)\tilde{x_2}+(1,0.5-0.5\gamma,0.5-0.5\gamma)}\end{array}\right\} \quad (2)$$

Subject to:

$$-(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{x_1}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{x_2} \le -(1,0.5-0.5\gamma,0.5-0.5\gamma)$$
$$(2,1-\gamma,1-\gamma)\tilde{x_1}+(3,1-\gamma,1-\gamma)\tilde{x_2} \le (15,1-\gamma,1-\gamma)$$
$$-(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{x_1} \le -(3,1-\gamma,1-\gamma)$$
$$\tilde{x_1},\tilde{x_2} \ge 0, \gamma \in [0,1]$$

Solving $\tilde{z_1}(\tilde{x})$ using variable transformation [1] to find a good fuzzy decision point:

$$\max \tilde{z_1}(\tilde{x}) = -(3,1-\gamma,1-\gamma)\tilde{y_1}+(2,1-\gamma,1-\gamma)\tilde{y_2}+(0,1-\gamma,1-\gamma)\tilde{w}$$

Subject to:

$$-(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_1}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_2}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{w} \le 0$$
$$(2,1-\gamma,1-\gamma)\tilde{y_1}+(3,1-\gamma,1-\gamma)\tilde{y_2}-(15,1-\gamma,1-\gamma)\tilde{w} \le 0$$
$$-(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_1}+(3,1-\gamma,1-\gamma)\tilde{w} \le 0$$
$$(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_1}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_2}+(3,1-\gamma,1-\gamma)\tilde{w} = 1$$
$$\gamma \in [0,1], \tilde{y_1},\tilde{y_2},\tilde{w} \ge 0$$

The fuzzy optimal solution are:

$$\tilde{y_1}=(0.3913,1-\gamma,1-\gamma), \quad \tilde{y_2}=(0.2826,1-\gamma,1-\gamma), \quad \tilde{w}=(0.1087,1-\gamma,1-\gamma)$$

Using the transformation we get:

$$\tilde{x_1}=(3.5998,1-\gamma,1-\gamma); \quad \tilde{x_2}=(2.5998,1-\gamma,1-\gamma);$$

$\max \tilde{z_1}(\tilde{x}) = (-0.625,1-\gamma,1-\gamma)$
Similarly for $\tilde{z_2}(\tilde{x})$:

$$\max \tilde{z_2}(\tilde{x}) = (7,1-\gamma,1-\gamma)\tilde{y_1}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_2}+(0,1-\gamma,1-\gamma)\tilde{w}$$

Subject to:

$$-(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_1}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_2}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{w} \le 0$$
$$(2,1-\gamma,1-\gamma)\tilde{y_1}+(3,1-\gamma,1-\gamma)\tilde{y_2}-(15,1-\gamma,1-\gamma)\tilde{w} \le 0$$
$$-(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{y_1}+(3,1-\gamma,1-\gamma)\tilde{w} \le 0$$
$$(5,1-\gamma,1-\gamma)\tilde{y_1}+(2,1-\gamma,1-\gamma)\tilde{y_2}+(1,0.5-0.5\gamma,0.5-0.5\gamma)\tilde{w} = 1$$
$$\gamma \in [0,1], \tilde{y_1},\tilde{y_2},\tilde{w} \ge 0$$

The fuzzy optimal solution is:

$$\tilde{y_1}=(0.1948,1-\gamma,1-\gamma), \quad \tilde{y_2}=(0,1-\gamma,1-\gamma), \quad \tilde{w}=(0.026,1-\gamma,1-\gamma)$$

By using the transformation we get:

$$\tilde{x_1} = (7.4923, 1-\gamma, 1-\gamma); \quad \tilde{x_2} = (0, 1-\gamma, 1-\gamma); \quad \max \tilde{z_2}(\tilde{x}) = (1.15, 1-\gamma, 1-\gamma),$$

By expanding first-order Taylor series approximation for objective functions $\tilde{z_1}$ and $\tilde{z_2}$ about fuzzy decision points $((3.5998, 1-\gamma, 1-\gamma); (2.5998, 1-\gamma, 1-\gamma))$ and $((7.4923, 1-\gamma, 1-\gamma); (0, 1-\gamma, 1-\gamma))$ in feasible region, respectively are obtained from:

$$\begin{aligned}\tilde{z}_1(\tilde{x}) = (-0.625, 1-\gamma, 1-\gamma) + (\tilde{x}_1 - (3.5998, 1-\gamma, 1-\gamma))\\ \cdot \frac{\partial \tilde{z}_1((3.5998, 1-\gamma, 1-\gamma);\ (2.5998, 1-\gamma, 1-\gamma))}{\partial \tilde{x}_1}\\ + (\tilde{x}_2 - (2.5998, 1-\gamma, 1-\gamma)) \cdot \frac{\partial \tilde{z}_1((7.4923, 1-\gamma, 1-\gamma);\ (0, 1-\gamma, 1-\gamma))}{\partial \tilde{x}_2}\end{aligned}$$

$$\tilde{z}_1(\tilde{x}) = (-0.2599)\tilde{x}_1 + (0.2835)\tilde{x}_2 - (0.4104, 1-\gamma, 1-\gamma)$$

Similarly:

$$\begin{aligned}\tilde{z}_2(\tilde{x}) = (1.15, 1-\gamma, 1-\gamma) + (\tilde{x}_1 - (7.4923, 1-\gamma, 1-\gamma))\\ \cdot \frac{\partial \tilde{z}_2(7.4923, 1-\gamma, 1-\gamma); (0, 1-\gamma, 1-\gamma)}{\partial \tilde{x}_1}\\ + (\tilde{x}_2 - (0, 1-\gamma, 1-\gamma)) \cdot \frac{\partial \tilde{z}_2(7.4923, 1-\gamma, 1-\gamma); (0, 1-\gamma, 1-\gamma)}{\partial \tilde{x}_2}\end{aligned}$$

$$\tilde{z}_2(\tilde{x}) = (0.0047)\tilde{x}_1 - (0.0448)\tilde{x}_2 - (1.3282, 1-\gamma, 1-\gamma)$$

The obtained FFMOLPP is equivalent to the following FFLPP when weights of objective functions are equal,

$$\max \tilde{z} = \tilde{z_1} + \tilde{z_2} = (0.1028\tilde{x_1} + 0.065\tilde{x_2} + 1.807, 1-\gamma, 1-\gamma)$$

Subject to same constraints, we get the fuzzy optimal solution as:

$$\tilde{x_1} = (3, 1-\gamma, 1-\gamma); \tilde{x_2} = (2, 1-\gamma, 1-\gamma); \max \tilde{z}(\tilde{x}) = (0.6299, 1-\gamma, 1-\gamma)$$

That is, the optimal solution of the FMOLFPP is given by:

$$\tilde{x}_1 = (a_1, a_2, a_3) = (2+\gamma,\ 3,\ 4-\gamma), \quad \tilde{x}_2 = (b_1, b_2, b_3) = (1+\gamma,\ 2,\ 3-\gamma)$$

with $\tilde{z}(\tilde{x}) = (-0.3701+\gamma,\ 0.6299,\ 1.6299-\gamma)$. The proposed method generates distinct fuzzy Pareto optimal solutions for different values of $\gamma \in [0, 1]$ in the considered FFMOLFPP. The corresponding fuzzy optimal solutions for each γ value are summarized in Table 2.

Table 2. Optimal solutions corresponding to various γ-cuts

γ	$\tilde{x_1}$	$\tilde{x_2}$	$\tilde{z}(\tilde{x})$
0	$(2, 3, 4)$	$(1, 2, 3)$	$(-0.3701,\ 0.6299,\ 1.6299)$
0.25	$(2.25, 3, 3.75)$	$(1.25, 2, 2.75)$	$(-0.1201,\ 0.6299,\ 1.3799)$
0.5	$(2.5, 3, 3.5)$	$(1.5, 2, 2.5)$	$(0.1299,\ 0.6299,\ 1.1299)$
0.75	$(2.75, 3, 3.25)$	$(1.75, 2, 2.25)$	$(0.3799,\ 0.6299,\ 0.8799)$
1	$(3, 3, 3)$	$(2, 2, 2)$	$(0.6229,\ 0.6299,\ 0.6299)$

At $\gamma = 1$, the solution yields $\tilde{x}_1 = 3$, $\tilde{x}_2 = 2$, and $\tilde{z}(\tilde{x}) = 0.6299$, which matches the crisp optimal result reported by Arya *et al.* [5] and Güzel and Sivri [18], corresponding to the fuzzy optimal solution in this study.

Example 2: a real-life application [5]

A university intends to launch distance learning centres (DLC) in cities A and B with the objective to enhance access to postgraduate education through distance learning initiatives. Due to financial constraints, the university cannot build centres indiscriminately across both cities. The primary goal is to determine how many centres can be established in each location while balancing budget, manpower and service delivery.

The fuzzy data required for planning includes fuzzy decision variables, budget requirements, manpower, number of students served, and fixed maintenance costs. This information is presented in Table 3.

Table 3. Decision support for DLC placement using fuzzy data

DLC	Fuzzy decision variable	Fuzzy budget requirement (in million $)	Approximate fuzzy number of students served (in thousand)	Approximate suzzy man power	Fixed approximate fuzzy maintenance cost (in million $)
City A	$\tilde{x_1}$	(1, 2, 3)	(2,3,4)	(2, 3, 4)	(1, 2, 3)
City B	$\tilde{x_2}$	(2, 3, 4)	(4, 5, 6)	(2,3,4)	(2, 4, 6)

Maximize the manpower-to-investment ratio: the university intends to create DLCs that offer direct access to students while ensuring a favourable ratio of user satisfaction to investment expenditure. This involves maximizing the ratio of the approximate number of required manpower to the total fuzzy investment budget. The objective is to ensure efficient deployment of human resources in line with the financial constraints. This goal is modeled using the following fuzzy fractional objective function:

$$\tilde{z_1}(\tilde{x}) = \max\left\{\frac{\text{Approximate number of man power}}{\text{Total investment approximate budget}}\right\}$$

$$\tilde{z_1}(\tilde{x}) = \max\left\{\frac{(2,3,4)\,\tilde{x_1} + (2,3,4)\,\tilde{x_2}}{(1,2,3)\,\tilde{x_1} + (2,3,4)\,\tilde{x_2} + (1,2,3)}\right\}$$

Maximize the student-service-to-investment ratio: the second objective focuses on maximizing service delivery, i.e., the number of students served per unit of investment. The university aims to determine the optimal number of centres to serve as many students as possible under budget limitations. This is expressed as:

$$\tilde{z_2}(\tilde{x}) = \max\left\{\frac{\text{Approximate number of service students}}{\text{Total investment approximate budget}}\right\}$$

$$\tilde{z_2}(\tilde{x}) = \max\left\{\frac{(2,3,4)\,\tilde{x_1} + (4,5,6)\,\tilde{x_2}}{(1,2,3)\,\tilde{x_1} + (2,3,4)\,\tilde{x_2} + (3,4,5)}\right\}$$

Given the scarcity of available resources, the university must satisfy the following fuzzy constraints:

1) Manpower constraint: $(2,3,4)\,\tilde{x_1} + (1,5,3)\,\tilde{x_2} \leq (20,25,30)$
   (The total approximate manpower must not exceed 25 persons.)
2) Budget constraint: $(1,2,3)\,\tilde{x_1} + (2,3,4)\,\tilde{x_2} \leq (5,10,15)$
   (The overall investment budget is capped at approximately $10 million.)

Therefore:

$$\max \tilde{z_1}(\tilde{x}) = \frac{(2,3,4)\tilde{x_1} + (2,3,4)\tilde{x_2}}{(1,2,3)\tilde{x_1} + (2,3,4)\tilde{x_2} + (1,2,3)}$$
$$\max \tilde{z_2}(\tilde{x}) = \frac{(2,3,4)\tilde{x_1} + (4,5,6)\tilde{x_2}}{(1,2,3)\tilde{x_1} + (2,3,4)\tilde{x_2} + (3,4,5)}$$

Subject to:

$$(2,3,4)\tilde{x_1} + (2,3,4)\tilde{x_2} \leq (20,25,30)$$
$$(1,2,3)\tilde{x_1} + (2,3,4)\tilde{x_2} \leq (5,10,15)$$
$$\tilde{x_1}, \tilde{x_2} \geq 0$$

Solution: the parametric form of the given FFM0LFPP is given by:

$$\max \tilde{z_1}(\tilde{x}) = \frac{(3,1-\gamma,1-\gamma)\tilde{x_1} + (3,2-2\gamma,2-2\gamma)\tilde{x_2}}{(2,1-\gamma,1-\gamma)\tilde{x_1} + (3,1-\gamma,1-\gamma)\tilde{x_2} + (2,1-\gamma,1-\gamma)}$$

$$\max \tilde{z_2}(\tilde{x}) = \frac{(3,1-\gamma,1-\gamma)\tilde{x_1} + (5,1-\gamma,1-\gamma)\tilde{x_2}}{(2,1-\gamma,1-\gamma)\tilde{x_1} + (3,1-\gamma,1-\gamma)\tilde{x_2} + (4,1-\gamma,1-\gamma)}$$

Subject to:

$$(3,1-\gamma,1-\gamma)\tilde{x_1} + (3,1-\gamma,1-\gamma)\tilde{x_2} \leq (25,5-5\gamma,5-5\gamma)$$
$$(2,1-\gamma,1-\gamma)\tilde{x_1} + (3,1-\gamma,1-\gamma)\tilde{x_2} \leq (10,5-5\gamma,5-5\gamma)$$
$$\gamma \in [0,1], \tilde{x_1}, \tilde{x_2}, \geq 0$$

Using variable transformation [1] to find a fuzzy decision point for objective functions $\tilde{z_1}(\tilde{x})$, we get:
$\tilde{x_1} = (5.1444,\ 1-\gamma,\ 1-\gamma);\quad \tilde{x_2} = (0,\ 1-\gamma,\ 1-\gamma);\quad \max \tilde{z_1}(\tilde{x}) = (1.25,\ 1-\gamma,\ 1-\gamma)$
For $\tilde{z_2}(\tilde{x})$: $\tilde{x_1} = (5.1444,\ 1-\gamma,\ 1-\gamma);\quad \tilde{x_2} = (0,\ 1-\gamma,\ 1-\gamma);\quad \max \tilde{z_2}(\tilde{x}) = (1.0714,\ 1-\gamma,\ 1-\gamma)$

By expanding first-order Taylor series approximation for objective functions $\tilde{z}_1$ and $\tilde{z}_2$ about fuzzy decision points $((5.1444, 1-\gamma, 1-\gamma); (0, 1-\gamma, 1-\gamma))$ and $((5.1444, 1-\gamma, 1-\gamma); (0, 1-\gamma, 1-\gamma))$ in feasible region, respectively are obtained from:

$$\begin{aligned}\tilde{z_1}(\tilde{x}) &= (1.25,1-\gamma,1-\gamma) + (\tilde{x_1} - (5.1444,1-\gamma,1-\gamma))(0.0416,1-\gamma,1-\gamma)\\ &\quad + (\tilde{x_2} - (0,1-\gamma,1-\gamma))(-0.0625,1-\gamma,1-\gamma)\\ &= (0.0416\tilde{x_1} - 0.0625\tilde{x_2} + 1.0417,1-\gamma,1-\gamma)\end{aligned}$$

$$\begin{aligned}\tilde{z_2}(\tilde{x}) &= (1.0714,1-\gamma,1-\gamma) + (\tilde{x_1} - (5.1444,1-\gamma,1-\gamma))(0.0612,1-\gamma,1-\gamma)\\ &\quad + (\tilde{x_2} - (0,1-\gamma,1-\gamma))(0.1275,1-\gamma,1-\gamma)\\ &= (0.0612\tilde{x_1} + 0.1275\tilde{x_2} + 10.7653,1-\gamma,1-\gamma)\end{aligned}$$

The obtained FFMOLPP is equivalent to the following FFLPP when weights of objective functions are equal $\max \tilde{z} = \tilde{z_1} + \tilde{z_2} = (0.1028\tilde{x_1} + 0.065\tilde{x_2} + 1.807, 1-\gamma, 1-\gamma)$ subject to same constraints ,we get the optimal solution as $\tilde{x_1} = (5.1444, 1-\gamma, 1-\gamma),\quad \tilde{x_2} = (0, 1-\gamma, 1-\gamma),\quad \max \tilde{z}(\tilde{x}) = (2.321, 1-\gamma, 1-\gamma)$

This demonstrates the efficiency of the proposed algorithm in quickly obtaining the optimal solution of the FFMOLFPP while accounting for both left and right fuzziness. $\tilde{x_1} = (4.1444+\gamma, 5.1444, 6.1444-\gamma)$ $\tilde{x_2} = (-1+\gamma, 0, 1-\gamma),\quad \max \tilde{z}(\tilde{x}) = (1.321+\gamma, 2.321, 3.321-\gamma)$. Table 4 gives the fuzzy optimal solution of the FMOLFPP for different values of γ.

Table 4. Optimal solutions corresponding to various γ-cuts

γ	$\tilde{x_1}$	$\tilde{x_2}$	$\tilde{z}(\tilde{x})$
0	(4.1444,5.1444,6.1444)	(-1,0,1)	(1.321, 2.321, 3.321)
0.25	(4.3944, 5.1444, 5.8944)	(-0.75,0,0.25)	(1.571, 2.321, 3.071)
0.5	(4.6444, 5.1444, 5.6444)	(-0.5,0,0.5)	(1.821, 2.321, 2.821)
0.75	(4.8944,5.1444,5.3944)	(-0.25,0,0.25)	(2.071, 2.321, 2.571)
1	(5.1444,5.1444,5.1444)	(0,0,0)	(2.321, 2.321, 2.321)

When $\gamma = 1$, we see that $\tilde{x_1} = 5.1444,\quad \tilde{x_2} = 0,\quad \max \tilde{z}(\tilde{x}) = 2.321$. This solution is same as crisp optimal solution. When $\gamma = 0$, the obtained values are $\tilde{x}_1 = (4.1444,\ 5.1444,\ 6.1444)$, $\tilde{x}_2 = (-1,\ 0,\ 1)$, and $\max \tilde{z}(\tilde{x}) = (1.321,\ 2.321,\ 3.321)$. This result coincides with the fuzzy optimal solution reported by Arya *et al.* [5]. After defuzzification of the obtained solution, the total number of DLCs in cities A and B is approximately 4 and 1, respectively.

5. RESULTS AND DISCUSSION

The proposed method differs from existing FFMOLFPP approaches by preserving fuzziness throughout the solution process, thereby avoiding the information loss typically caused by defuzzification. By generating fuzzy Pareto optimal solutions across varying γ-levels, it offers a clearer view of the trade-off between uncertainty and crisp decision-making. Tables 2 and 4 show that for $\gamma = 0$, the widest uncertainty bands are obtained in both decision variables and objective values, while increasing γ gradually narrows the fuzziness and converges to a crisp solution at $\gamma = 1$. For example 1, the solution at $\gamma = 1$ is $\tilde{x}_1 = 3$, $\tilde{x}_2 = 2,\quad \max \tilde{z}(\tilde{x}) = 0.6299$ matches the crisp optimum reported by Arya *et al.* [5] and Güzel and Sivri [18]. Similarly, for example 2, the solution at $\gamma = 1$ is $\tilde{x}_1 = 5.1444,\quad \tilde{x}_2 = 0,\quad \max \tilde{z}(\tilde{x}) = 2.321$ also coincides with Arya *et al.* [5]. The γ-sensitivity analysis highlights that lower γ-values capture broader uncertainty and flexibility, whereas higher γ-values provide more precise but less fuzzy solutions. This shows that people who make decisions can choose solutions based on how much risk and uncertainty they can handle. The suggested technique keeps the complete spectrum of uncertainty and prevents premature defuzzification, which makes it easier to understand and more realistic than past works.

Lastly, the Taylor series linearization is computationally efficient since it converges quickly and treats left and right spreads equally. This research looks at discrete γ-levels and assumes triangular fuzziness. However, the framework may be used with continuous γ-levels and other types of fuzzy forms. These results bolster the method's relevance to practical decision-making scenarios characterized by inherent uncertainty, including resource allocation and educational planning.

6. CONCLUSION

This paper proposes an effective method for solving FFMOLFPPs that involve uncertainty in both objective functions and constraints. By integrating variable transformation, the γ-parametric representation of fuzzy numbers, and the first-order Taylor series approximation, the approach successfully reformulates fuzzy fractional objectives into a linear programming framework,yielding optimal solutions expressed in fuzzy terms, without the need to defuzzify or convert the problem into a crisp equivalent.The proposed technique enables the derivation of both crisp and fuzzy optimal solutions at various γ-levels. At $\gamma = 1$, the solution converges to a crisp optimal result, while at lower γ-levels, the model captures the full range of fuzziness, producing fuzzy-valued decision variables and objectives

Overall, the proposed approach provides a robust and adaptable framework for addressing decision-making problems under uncertainty. It enables decision-makers to explore multiple solution scenarios based on their tolerance for fuzziness, thereby improving both the practical applicability and interpretability of the optimization outcomes in real-world contexts.

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Anitha Ganesan	✓	✓	✓		✓	✓		✓	✓		✓			
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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal Analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject Administration
Fu : **Fu**nding Acquisition

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The authors declare that they have no conflict of interest.

ETHICAL APPROVAL

This article does not contain any studies with human participants or animals performed by any of the authors.

DATA AVAILABILITY

The data supporting the findings of this study are openly available at references number [5] and [18].

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APPENDIX

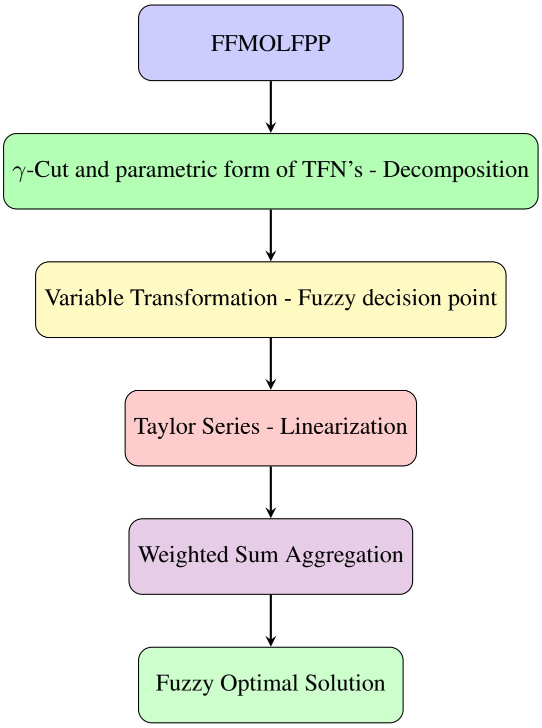

Figure 1. Flowchart of the proposed FFMOLFPP methodology

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