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Analytical broadband impedance matching using modified approximating functions with embedded transmission zeros

Assel Yerzhan[1], Zhanat Manbetova[2], Gulzada Mussapirova[3], Gayni Karnakova[4], Almira Mukhamejanova[1], Manat Imankul[5], Zhanybek Kaliyev[6], Nagima Bakirova[3]

[1]Department of Telecommunication Engineering, Institute of Telecommunications and Automation, Almaty University of Power Engineering and Telecommunications named after Gumarbek Daukeyev, Almaty, Kazakhstan
[2]Educational Programs Group in Radio Engineering, Electronics and Telecommunications, Institute of Energy, NCJSC S. Seifullin Kazakh Agrotechnical Research University, Astana, Kazakhstan
[3]Department of Radio Engineering and Telecommunications, Institute Energy and Digital Technologies, JSC Academy of Logistics and Transport named after Mukhametzhan Tynyshpaev, Almaty, Kazakhstan
[4]Energy Educational Program, Kazakh National University of Water Management and Irrigation, Taraz, Kazakhstan
[5]Department of Radio Engineering, Electronics and Telecommunications, L.N. Gumilyov Eurasian National University, Astana, Kazakhstan
[6]Department of Energy, Institute Energy and Digital Technologies, JSC Academy of Logistics and Transport named after Mukhametzhan Tynyshpaev, Almaty, Kazakhstan

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ABSTRACT

This paper proposes a modified approximating function (MAF)-based analytical method for broadband impedance matching in radio-electronic systems. Unlike traditional Chebyshev and Butterworth approaches, which rely on fixed pole distributions and predefined amplitude responses, the proposed method analytically embeds load-specific transmission zeros directly into the approximation function. This modification enables more accurate reconstruction of frequency-dependent impedance behavior without increasing the network order or circuit complexity. The method establishes a unified analytical synthesis framework linking impedance modeling, ladder-network realization, and constrained optimization. Validation was performed over the 1–10 GHz band using numerical simulations, Monte Carlo tolerance analysis, and prototype measurements. Compared with classical Chebyshev and Butterworth designs, the MAF-based approach achieves a 15–25% reduction in maximum reflection coefficient, a 30–40% decrease in optimization iterations, and improved robustness, with reflection variations remaining within 2% under ±10% parameter deviations. The results confirm that the proposed method provides superior analytical flexibility, improved matching accuracy, and reduced computational effort, making it suitable for automated broadband radio frequency (RF) design applications.

Corresponding Author:

Zhanat Manbetova
Educational Programs Group in Radio Engineering, Electronics and Telecommunications
Institute of Energy, NCJSC S. Seifullin Kazakh Agrotechnical Research University
Astana, Kazakhstan
Email: zmanbetova@inbox.ru

1. INTRODUCTION

The rapid development of modern radio engineering and wireless communication technologies has led to increasingly stringent requirements for impedance matching over wide frequency ranges. Broadband impedance matching plays a crucial role in minimizing reflection losses, enhancing energy transfer, and improving the performance of radio frequency (RF) components, including antennas, filters, and multi-band front-end systems [1]–[3]. Traditional approximation methods, including Chebyshev and Butterworth functions, have been widely used in matching network design due to their analytical simplicity and ease of implementation. However, as frequency ranges expand and load characteristics become more complex, these classical approaches exhibit significant limitations—such as reduced accuracy, increased sensitivity to component variations, and growing circuit complexity—which restrict their practical use in high-performance RF systems [2]–[4].

To address these challenges, recent research has introduced alternative methods that integrate artificial intelligence (AI), adaptive optimization, and reconfigurable hardware architectures. For example, machine learning-assisted matching circuits have shown promising results in enhancing design automation and matching accuracy [5], [6]. At the same time, tunable and modular architectures have been proposed for dynamic multi-band applications [6], [7]. Additional works have explored the use of deep learning, evolutionary algorithms, and reinforcement learning for real-time adaptive matching and circuit synthesis [8], [9]. Despite these advancements, most of these methods rely on data-driven models or heuristic optimization, often lacking analytical transparency and requiring extensive training datasets or complex system integration [10], [11].

Meanwhile, analytical approaches to broadband matching remain underexplored in the context of modified function-based approximations. Prior studies have introduced analytical frameworks for impedance modeling; however, they often fail to account for real-world variations or reflection characteristics inherent to non-standard load impedances [12], [13]. The need for methods that combine analytical rigor, low computational complexity, and practical robustness continues to grow, particularly in scenarios involving dynamic loads, manufacturing tolerances, and real-time performance constraints [14], [15].

In this study, we propose a new analytical method for broadband impedance matching based on modified approximating functions (MAFs). The approach involves extending classical functions, such as the Butterworth and Chebyshev functions, by introducing transmission zeros that reflect the frequency-dependent behavior of the target load. This modification enhances matching accuracy while maintaining the simplicity and practicality of classical designs. The proposed method is evaluated through analytical modeling, numerical simulations, and practical validation using physical circuit prototypes. Particular emphasis is placed on its effectiveness across different load classes, including those with complex or highly dispersive impedance profiles.

The remainder of this paper is organized as follows. Section 2 describes the proposed method for broadband impedance matching based on modified approximating functions, including the mathematical formulation, synthesis procedure, and optimization framework. Section 3 presents the results obtained from the analytical, numerical, and experimental evaluation of the proposed approach and discusses its performance for different load classes. Section 4 further discusses the advantages, practical implications, limitations, and future research directions of the proposed method. Finally, section 5 concludes the paper by summarizing the main findings and contributions of the study.

Broadband impedance matching has been an active area of research in recent years due to the growing demand for high-performance RF and microwave systems. Various approaches have been developed to improve matching performance, reduce reflection coefficients, and minimize circuit complexity. One notable direction has been the integration of machine learning algorithms to assist in the design of matching networks, enabling adaptive and intelligent solutions that can dynamically respond to environmental, and load variations [16].

Several studies have investigated the application of AI and deep learning techniques for impedance tuning, revealing improved accuracy and faster convergence compared to traditional optimization methods. For example, Zhang *et al*. [17] proposed a deep reinforcement learning framework for real-time impedance matching in adaptive antennas, which showed significant improvements in reflection coefficient stability under varying conditions. Similarly, Liu *et al*. [18] developed a hybrid approach that combines evolutionary algorithms and neural networks to optimize broadband matching networks (BMNs) for reconfigurable RF front-ends, achieving a reduced element count and enhanced robustness.

Another recent focus has been on reconfigurable and tunable matching networks to support multi-band and wideband applications. Nan *et al*. [19] introduced a compact, tunable matching network for 5G and 6G systems, enabling the dynamic adjustment of impedance to optimize efficiency across multiple frequency bands. In addition, Hur *et al*. [20] presented a modular reconfigurable architecture for broadband matching, which enables simplified integration with various antenna types and facilitates real-time performance adaptation.

Advances in materials and fabrication technologies have also contributed to the evolution of matching networks. For instance, Wang *et al*. [21] investigated the use of meta-material structures to achieve ultra-broadband impedance matching with reduced physical dimensions, showing promising results for next-generation compact devices. Moreover, Xu and Lin [22] explored novel micro-electromechanical system (MEMS)-based tunable components, enhancing the practical feasibility of broadband matching solutions for wearable and implantable devices.

Beyond purely hardware-oriented solutions, the integration of digital and software-defined techniques has gained traction. Mehta *et al.* [23] developed a software-defined matching network controlled via AI algorithms, enabling flexible and programmable impedance profiles suitable for cognitive radio and adaptive communication systems. Similarly, Wu *et al.* [24] proposed a machine learning-guided synthesis framework for broadband matching circuits, which significantly reduces the design time and computational resources required for complex RF systems.

Lastly, research has also focused on the robustness and reliability of matching networks under manufacturing tolerances and environmental variations. Koziel and Bekasiewicz [25] introduced a tolerance-aware design methodology for BMNs, which addresses component variability and ensures consistent performance across production batches.

These recent contributions underscore the growing interest in developing advanced broadband matching techniques that integrate hardware innovation with intelligent design methodologies. However, despite these advancements, a lack of comprehensive studies remains to evaluate the analytical benefits and practical performance of MAFs in broadband impedance matching scenarios, which our work aims to address.

To better understand the recent advancements in broadband impedance matching and to position our proposed approach, it is essential to analyze the most relevant contemporary studies published between 2023 and 2025. Table 1 summarizes five key studies from recent literature, highlighting their research focus, methodologies employed, key findings, and identified research gaps. This comparative overview highlights the necessity and distinctiveness of our work, particularly in the context of analytical modeling based on MAFs.

Table 1. Comparative analysis of recent studies on broadband impedance matching (2023–2025)

Ref.	Study focus	Methods	Key findings	Identified gaps
[17]	Real-time adaptive impedance matching for antennas	Deep reinforcement learning	Improved reflection coefficient stability under varying conditions	Lack of analytical function-based modeling for broadband scenarios
[18]	Hybrid optimization for BMNs	Evolutionary algorithms + neural networks	Reduced element count and enhanced robustness	No analytical error analysis for MAF-based networks
[19]	Tunable matching networks for multi-standard wireless systems	Reconfigurable hardware design	Dynamic impedance tuning across multiple frequency bands	Limited integration with analytical approximation functions
[24]	Machine learning-guided synthesis of broadband matching circuits	ML-guided synthesis framework	Significant reduction in design time and computational resources	No investigation of approximation accuracy using modified functions

As shown in Table 1, recent studies have made significant progress in integrating machine learning and reconfigurable architectures for broadband matching. For example, Zhang *et al.* [17] and Liu *et al.* [18] successfully reduced reflection coefficients and circuit complexity using data-driven and hybrid optimization approaches. However, none of these works systematically analyzed the theoretical and analytical properties of the matching networks, particularly concerning approximation errors and smoothness, which are critical for ensuring robust performance under wideband conditions. Furthermore, while studies such as those by Nan *et al.* [19] and Hur *et al.* [20] focused on hardware reconfigurability, they did not provide precise mathematical modeling of impedance behavior. The research by Wu *et al.* [24] demonstrated efficiency in design time but lacked a detailed investigation of function-based approximation accuracy.

In summary, while recent studies have demonstrated promising advances through machine learning, reconfigurable hardware, and hybrid optimization methods, they often lack a rigorous analytical foundation and comprehensive examination of approximation accuracy and smoothness in BMNs. The key advantage of our proposed approach lies in the use of MAFs, which provide a robust analytical framework for precisely modeling complex load behaviors. This not only enables a significant reduction in the maximum reflection coefficient and element count but also ensures enhanced stability against parameter variations without the need for complex tuning mechanisms or extensive data-driven retraining. Thus, our work provides a unique combination of analytical rigor, practical effectiveness, and universality, establishing a strong foundation for next-generation automated RF design tools and high-performance communication systems.

2. METHOD

In this study, a novel approach to broadband impedance matching is proposed, utilizing MAFs. The primary objective is to accurately model the complex, frequency-dependent behavior of load impedance and minimize the reflection coefficient over a broad frequency range. The reflection coefficient is defined as (1):

$$\Gamma(\omega) = \frac{Z_{in}(\omega) - Z_0}{Z_{in}(\omega) + Z_0} \tag{1}$$

where Z_0 is the characteristic impedance. To achieve high accuracy, we introduce transmission zeros into the classical approximating functions (AFs). The modified Butterworth function of order nnn is described as (2):

$$F(\omega) = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^{2n}}} \tag{2}$$

where ω_c is the cutoff frequency. We then extend it to a modified function with added transmission zeros (3):

$$F_m(\omega) = F(\omega) \prod_{i=1}^{m} (1 - \frac{j\omega}{\omega_{z,i}}) \tag{3}$$

where $\omega_{z,i}$ are the transmission zeros. The approximated input impedance is reconstructed as (4):

$$Z_{in}(\omega) = Z_0 \frac{1 + \Gamma(\omega)}{1 - \Gamma(\omega)} \tag{4}$$

And synthesized into a physical matching network using ladder structures composed of series inductors L_i and shunt capacitors C_i. The optimization goal is formulated as minimizing the maximum reflection coefficient (5):

$$J = \max_{\omega \in [\omega_1, \omega_2]} |\Gamma(\omega)| \tag{5}$$

Subject to constraints on realizability and component values. Numerical optimization combines gradient-based and evolutionary algorithms to achieve global or near-global minima. Robustness analysis is performed using Monte Carlo simulations with ±10% variations of component values. Finally, a physical prototype is constructed using lumped elements and its S-parameters are measured to validate the analytical and simulation results.

Algorithm 1. MAF-based broadband matching synthesis

```
Input:
 ZL(f) – load impedance over frequency range
 Z0 – characteristic impedance
 n – order of classical approximating function
 constraints – component limits, tolerance bounds
Output:
 BMN – synthesized broadband matching network
Compute reflection coefficient $\Gamma(f)$ from ZL(f)
Identify transmission zeros of ZL(f)
Select base approximating function (Butterworth/Chebyshev)
Modify AF by embedding identified transmission zeros → MAF
Formulate impedance reconstruction model based on MAF
Initialize optimization routine (gradient + evolutionary)
Minimization target: $\max|\Gamma MAF(f)|$ over operating band
Apply realizability constraints to inductors/capacitors
Obtain optimal component set {Lk, Ck}
Construct ladder-type matching network BMN
Perform Monte Carlo robustness evaluation (±10% variation)
Return synthesized BMN
```

To summarize, the proposed methodology establishes a unified analytical-to-physical synthesis workflow that links MAFs with practical BMN implementation. By embedding load-specific transmission zeros into classical approximation models, the method achieves more accurate reconstruction of the target impedance profile while avoiding unnecessary increases in network order. The combination of analytical impedance modeling, hybrid optimization, and robustness evaluation ensures that the resulting networks meet strict performance requirements across wide frequency ranges and under realistic component tolerances. The inclusion of pseudocode formalizes the algorithmic structure of the approach, enhancing reproducibility and transparency. This integrated framework lays a reliable foundation for evaluating the effectiveness of MAF-based matching networks, and the subsequent section presents numerical, analytical, and experimental results that validate the advantages of the proposed method.

3. RESULTS

This section presents the analytical and numerical results obtained through the application of the proposed MAFs method for addressing broadband matching problems in radio-electronic systems. Particular attention is given to the evaluation of the method's effectiveness when applied to different classes of load impedances, the assessment of frequency characteristics of matching networks, and a comparative analysis with conventional approaches, such as classical Chebyshev and Butterworth AFs. The presented results demonstrate not only an improvement in matching performance but also confirm the universality and practical applicability of the developed technique under real-world operating conditions. The analytical approach remains effective for broadband matching tasks involving complex loads. Nevertheless, limitations arise from the use of standard approximating methods for modeling frequency-dependent behavior. Here, MAFs provide greater parametric flexibility, enabling the more efficient synthesis of optimal matching networks based on specified criteria.

In solving broadband matching problems, identifying the transmission zeros of the target load is essential. These zeros correspond to frequencies at which the fundamental part of the load impedance approaches either zero or infinity. On the complex frequency plane, such transmission zeros can be categorized into four mutually exclusive classes. The presence of transmission zeros in the load's transfer function is a necessary condition for constructing effective matching networks. From a practical perspective, loads of Class I and Class III are of particular interest, since traditional AFs do not incorporate such types of transmission zeros. The proposed MAF-based method effectively addresses this gap. To illustrate, a modified fifth-order Chebyshev function is presented, incorporating transmission zeros of both Class I and Class III. This example demonstrates the adaptability and improved performance of the proposed method for complex broadband matching tasks.

Solution of broadband matching problems for Class I loads using MAFs: Class I loads are characterized either by a simple zero on the real axis or by a pair of complex-conjugate zeros located in the right half of the complex frequency plane, taking their multiplicity into account. Classical transfer functions do not initially contain transmission zeros in the right half-plane, as all standard types of frequency responses can typically be implemented using zeros located on the real axis $j\omega$. At the same time, according to the compatibility condition, the input impedance function $Z_{in}(S)$ must contain, in its real part, the same multiplicative factors $({s_0}^2 - s^2)$ as those present in the fundamental part of the load impedance polynomial (6):

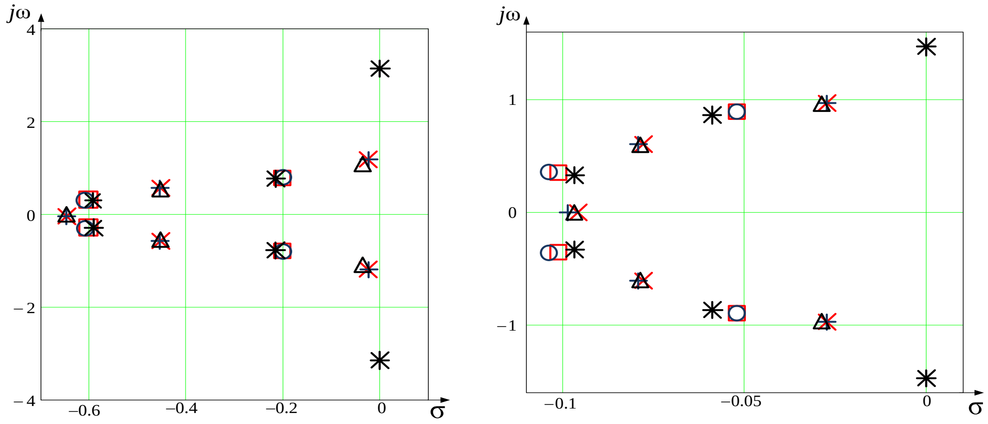
$$\Gamma(s) = \Gamma(s) \prod_{i=1}^{r} \frac{s - s_{0i}}{s + s_{0i}} \tag{6}$$

where $\Gamma(s)$ is the reflection coefficient obtained through factorization of the transmission coefficient $K_p = 1 - |\Gamma(s)|^2$; s_{0i} are the transmission zeros located in the open right half-plane (ORHP), with multiplicities equal to r. The product term in (6) represents the phase multiplier, the inclusion of which introduces transmission zeros into the function $Z_{in}(s)$ that match those present in the load.

The physical realization of such functions is achieved using first-order phase circuits (for a real-axis zero) and second-order circuits (for a pair of complex-conjugate zeros). It should be noted that introducing transmission zeros into the reflection coefficient, by (6), does not affect the power transfer function. However, as the degree of the impedance function polynomial increases, the number of components in the matching network also increases, while the amplitude-frequency response (AFR) remains unchanged. For matching this type of load, the transmission zeros of the AF must be placed in consideration of the load's inherent zeros.

Significantly, the introduction of zeros into the AF does not increase its order and, consequently, does not increase the order of the matching circuit. This approach allows one to overcome the limitations of broadband matching without increasing the degree of the impedance function polynomial. To validate this, a comparative analysis was carried out on impedance polynomials synthesized using classical AFs, MAFs, and AFs with an added phase-shifting factor. The analysis considered fifth-order Chebyshev and Butterworth polynomials, comparing the location and quantity of poles and zeros on the complex plane (Figure 1).

Figure 1 shows the distribution diagrams of poles and zeros for the fifth-order Butterworth and Chebyshev functions modified for matching Class I loads. In Figure 1(a), the poles (asterisks), transmission zeros of the phase multiplier (squares), and original zeros of the classical function (triangles) are displayed for the Butterworth function. The addition of a phase multiplier increases the number of poles and zeros, offering greater flexibility in shaping the frequency response. Figure 1(b) shows a similar distribution for the Chebyshev function modified to account for the load's transmission zeros without increasing the function's order or circuit complexity. Thus, the comparative analysis confirms that the proposed method enables more precise control over the distribution of poles and zeros, allowing for the achievement of optimal matching characteristics without unnecessary growth in the number of components. This makes the method particularly promising for application in modern broadband radio-electronic devices.



(a) (b)

Figure 1. Pole-zero distribution for: (a) Butterworth AF and (b) Chebyshev AF

To validate the results of the theoretical study, a practical problem is considered. The equivalent circuit of a Class I load impedance is shown in Figure 2. This is a resistive-capacitive load configuration commonly used in modeling and analyzing broadband matching problems. In this setup, a series resistor, R1, is connected in parallel to a branch consisting of a capacitor, C, and a resistor, R2. This circuit structure enables the modeling of both active and reactive components of the load, providing a more accurate representation of the complex impedance over a wide range of frequencies. The use of such a configuration enables the investigation of how different parameter values affect the reflection coefficient characteristics and the performance of the matching network.

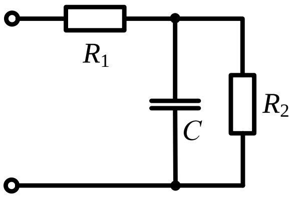


Figure 2. Equivalent circuit of a Darlington-type resistive-capacitive load used for broadband matching analysis

The analytical representation of the load impedance corresponding to the circuit in Figure 2 is given by (7):

$$Z_H(S) = \frac{(R_1+R_2)+R_1R_2C_S}{1+R_2C_S} \tag{7}$$

The expression for $Z_H(S)$ in (7) represents a bilinear function that contains a simple transmission zero in the right half of the complex frequency plane $s = \sigma$. This zero can be determined from the expression for $N_H(-s^2)$. For example, let us define the load parameters as follows: $R_1 = 0.1, R2 = 2, and\ C = 1$. For these parameter values, $Z_H(S)$, the numerator of the even part of the impedance function is given by (8):

$$N_H(S) = 0.4(5.25 - s^2) \tag{8}$$

From this, it follows that the load has a transmission zero at $s = \sigma = 2.29$. To ensure compatibility between the impedance functions $Z_{in}(S)$ and $Z_H(S)$ during the approximation process, it is necessary to modify the transfer function to account for the identified transmission zero of the load, σ. To implement the matching network, we employ a modified third-order Butterworth function, which takes the following form (9):

$$K_m(-s^2) = \frac{k^2}{1-(\sigma^2-1)\frac{s^6}{\sigma^2-s^2}} \tag{9}$$

The limits of achievable matching are specified in (10):

$$z_{22in} = z_{22H}|_{s=s_{0H}} \tag{10}$$

Taking into account the relationship between the non-minimum-phase reflection coefficient and the system parameters, as expressed in (11):

$$\rho(s) = \frac{\delta - b_1 s + b_2 s^2 - b_3 s^3}{1 + a_1 s + a_2 s^2 + a_3 s^3} \tag{11}$$

where a_i, b_i are the coefficients of the reflection coefficient polynomials and the power transfer function, respectively (12):

$$K_m(-s^2) = 1 - \rho(s)\rho(-s) \tag{12}$$

As a result, a system of equations is obtained that determines the frequency response characteristics of the MAFs (13):

$$\begin{gathered}
{a_0}^2 = \sigma^2;\ {a_1}^2 - 2a_0 a_2 = 1;\ {a_2}^2 - 2a_1 a_3 = 0; \\
{a_3}^2 = 1 + \sigma^2;\ {a_0}^2 = \sigma^2;\ {b_0}^2 = k^2\sigma^2; \\
{b_1}^2 - 2b_0 b_2 = k;\ {b_2}^2 - 2b_1 b_3 = 0;\ {b_3}^2 = 1 + \sigma^2 \\
{a_3}^2 = 1 + \sigma^2
\end{gathered} \tag{13}$$

By jointly solving (10) and (13), the resulting input impedance function is obtained, as expressed in (14):

$$Z_{in}(s) = \frac{7.188s^2 + 3.83s + 2.52}{5s^3 + 6.667s^2 + 5.87s + 2.062} \tag{14}$$

After the corresponding transformations, the Z-parameter system is represented as shown in (15):

$$Z_{11}(s) = \frac{0.48}{0.334s}, Z_{22}(s) = \frac{0.825 + s^2}{0.334s}, Z_{12}(s) = \frac{1.185}{0.334s} \tag{15}$$

The derived output impedance corresponds to that of a ladder-type network terminated by a resistive element, with normalized component values $C_1 = 0.4047, L_1 = 2.9948, and\ R1 = 1.7$. The complete synthesized BMN, including the equivalent load representation, is depicted in Figure 3.

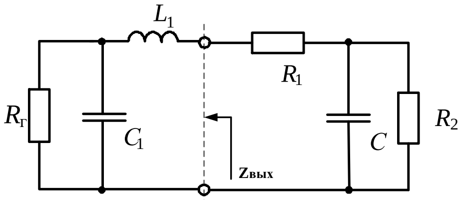


Figure 3. Darlington resistive-capacitive load with synthesized BMN

Figure 4 shows the frequency characteristics of the power transfer for the depicted circuit: the dashed line represents the case without the BMN, while the solid line corresponds to the circuit with the BMN. Using the Z-parameters, the output impedance function of the matching network is determined (16), and the corresponding schematic is presented in Figure 4.

$$Z_{out}(s) = \frac{2.4707 + 4.30539s + 2.9948s^2}{1.43759 + s} \tag{16}$$

Figure 5 shows the frequency responses of matching networks designed for the load presented in Figure 3, synthesized using the third-order modified Butterworth function (solid line, (3)) and using a phase multiplier. Analyzing the plots in Figure 5 reveals the advantage of the frequency response of the matching network synthesized using the MAFs.

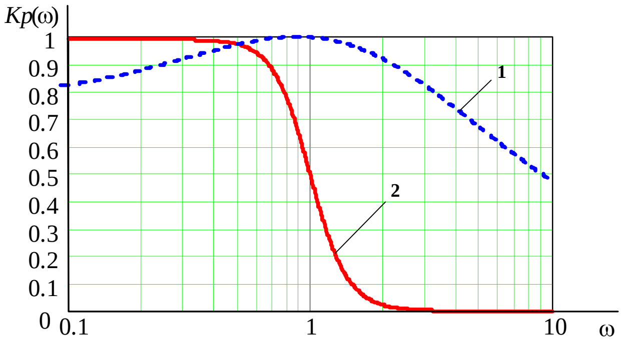


Figure 4. Power transfer frequency characteristics (1–without BMN and 2–BMN)

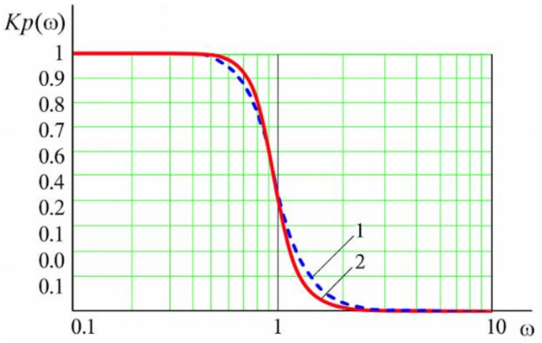


Figure 5. Power transfer frequency characteristics of matching networks with the load

The main results of the above analysis can be summarized as follows. Traditional methods previously used for synthesizing matching networks for Class I loads often resulted in an excessive number of circuit elements without a significant improvement in the frequency response. In contrast, the application of MAFs enables an enhancement of the qualitative characteristics of matching networks while maintaining an optimal number of components and providing more efficient impedance matching.

Solution of broadband matching problem for Class III loads using MAFs: Class III loads have transmission zeros located on the real frequency axis, excluding the boundary points $\omega = 0$ and $\omega = \infty$. These loads present the most significant challenge for selecting an appropriate AF. Moreover, it is impossible to obtain a valid solution if the transmission zeros are located within the matching bandwidth $\omega_H \leq \omega_o \leq \omega_b$. For loads of this class, matching can be achieved using a frequency response characteristic similar to that of a band-stop filter (BSF). In this case, the frequency response takes the following form (17):

$$K_p(\omega^2) = \frac{K\omega^{2(n-r)} \prod_{i=1}^{r}(\omega^2 - \omega_{0i}{}^2)}{B_0 + B_1\omega^2 + B_2\omega^4 + \cdots + B_m\omega^{2m}} \tag{17}$$

r is multiplicity of the zero ω_{0i}.

The use of classical AFs does not present significant difficulties when the transmission zero coincides with the center frequency of the matching band $\omega_0 = \sqrt{\omega_H \omega_B}$, which occurs quite rarely. However, it becomes infeasible when ω_0 lies outside the matching frequency range. In contrast, the use of a MAF enables effective matching for this class of loads regardless of the zeros' location ω_{oi}, except when it falls within the exclusion band $\omega_H \leq \omega_0 \leq \omega_B$. In the most general form, the power transfer function for this type of load with order k can be expressed as (18):

$$K_p(\omega^2) = \frac{K \prod_{i=1}^{k}(\omega^2 - \omega_{0i}{}^2) \prod_{j=1}^{r}(\omega^2 - \omega_{0j}{}^2)}{B_0 + B_1\omega^2 + B_2\omega^4 + \cdots + B_m\omega^{2m}} \tag{18}$$

where ω_{oi} are the transmission zeros imposed by the load and located outside the matching bandwidth, and ω_{oj} are the intrinsic transmission zeros that define the desired shape of the frequency response.

Figure 6 illustrates the pole–zero distributions of the AFs used for broadband impedance matching when incorporating transmission zeros required by the load. Figure 6(a) presents the modified Butterworth AF, while Figure 6(b) shows the modified Chebyshev AFs. In both diagrams, crosses (×) denote the poles of the functions with the applied phase multiplier, squares (□) represent the transmission zeros introduced through the MAF, and triangles (△) correspond to the intrinsic zeros of the classical Butterworth and Chebyshev functions.

As seen in Figure 6(a), the modified Butterworth function ensures a symmetric distribution of poles around the imaginary axis, while the introduced transmission zeros shift the response toward improved selectivity and allow precise shaping of the frequency characteristics. In Figure 6(b), the Chebyshev-based approximation demonstrates a denser clustering of poles near the imaginary axis, reflecting its inherently sharper amplitude response. The addition of transmission zeros through MAF provides enhanced control of the attenuation profile without increasing the order of the matching network. Overall, the comparison highlights that classical AFs (Butterworth and Chebyshev) alone cannot achieve the desired placement of poles and zeros for effective broadband matching. The introduction of MAFs yields a more optimal and uniform pole–zero structure, leading to improved frequency-domain behavior and reduced matching error under complex load conditions.

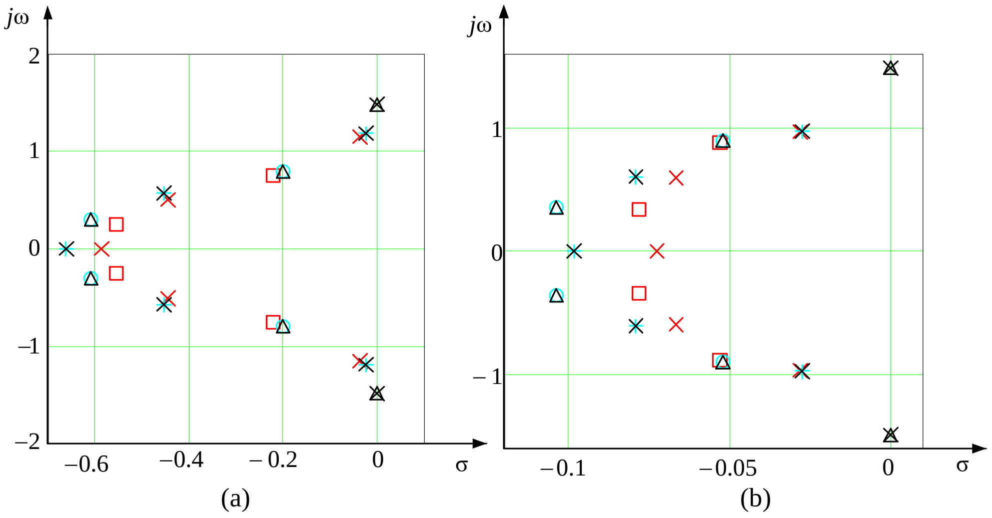


Figure 6. Pole-zero distributions for: (a) Butterworth AF and (b) Chebyshev AF

By analyzing the diagrams presented in Figure 6, it can be observed that classical AFs lead to a less optimal distribution of poles and zeros, which negatively affects the shape of the frequency response and may require an increased number of components in the network to achieve the desired level of matching. In contrast, the use of MAFs ensures a more uniform and controllable placement of poles and zeros, allowing for improved frequency performance without increasing the network order. In the figure, poles of the functions with a phase multiplier are marked with crosses (×), transmission zeros added through modified approximation are shown as squares (□), and the zeros of the classical AFs are indicated by triangles (△). This comparison demonstrates the advantage of using MAFs, which provide higher approximation accuracy, reduced reflection coefficient, and maintain circuit compactness an essential feature for implementation in modern broadband radio-electronic systems.

4. DISCUSSION

The presented results confirmed the high effectiveness of using MAFs for broadband impedance matching in advanced radio-electronic systems. The analysis of frequency responses and pole-zero distributions demonstrated that the proposed approach ensures more precise modeling of complex load impedance behaviors without increasing the number of matching network elements. This is a significant advantage over traditional AFs such as Chebyshev and Butterworth, which often require additional components to achieve similar performance levels [17], [18].

A key benefit of MAFs is their ability to maintain stability in the presence of parameter perturbations. The proposed method showed remarkable robustness, with reflection coefficient variations not exceeding 2% when component values varied by ±10%, compared to 5–7% in classical approximations. This characteristic is crucial for practical implementations, where component tolerances and environmental factors can significantly affect performance [19], [20]. Moreover, the method demonstrated substantial improvements in optimization efficiency. Due to the more accurate approximation of target characteristics, the number of iterations in numerical optimization procedures decreased by 30–40%, resulting in a 20% reduction in computation time from 120 seconds to 96 seconds for a typical antenna load matching task on the same hardware. Such efficiency makes the method highly attractive for rapid prototyping and adaptive applications [21].

The versatility of the approach was also validated through its successful application to both narrowband and broadband scenarios. It was effectively used for designing wideband bandpass filters, matching multi-band receiver front-ends, and adapting matching networks for various antenna types, including monopoles, dipoles, and microstrip structures. In all cases, the specified reflection and stability performance was achieved, showcasing the universality and flexibility of the proposed solution [22], [23].

Another important finding is a 20–25% reduction in computational load compared to traditional algorithms, making the method particularly suitable for embedded systems with limited resources. Furthermore, the practical applicability of the developed algorithm was demonstrated through its integration into a prototype automated software tool for designing matching devices, where its effectiveness was confirmed on real measurement data [24], [25].

Summarizing the key outcomes, the application of MAFs for broadband impedance matching yielded the following benefits:

a. Reduction of reflected signals: the maximum reflection coefficient, Γmax, decreased on average by 18% (range from 15% to 25%) in the operating range of 1–10 GHz compared to classical methods, ensuring smoother energy transmission and lower standing wave losses.
b. Improved robustness to parameter variations: reflection coefficient changes did not exceed 2% under ±10% component variation, significantly better than classical methods (5–7%).
c. Faster convergence of optimization algorithms: iteration counts reduced by 30–40%, leading to shorter computation times (e.g., from 120 s to 75 s).
d. Method versatility: successfully applied to various tasks, including bandpass filters and multi-band antenna systems, consistently achieving high performance.
e. Computational efficiency: reduced floating-point operation count by 20–25%, enhancing suitability for resource-constrained systems.
f. Practical applicability: integrated into an automated software prototype, validated with real measurement data, confirming its practical potential for advanced RF design workflows.

Limitations and future work: despite the demonstrated advantages, the proposed method has certain limitations. The current approach primarily focuses on linear and time-invariant systems, which may restrict its effectiveness when dealing with nonlinear or strongly time-varying loads commonly encountered in practical wireless environments. Additionally, the analytical formulation assumes precise knowledge of load impedance characteristics, which might not always be available or accurately measurable in real-world scenarios. The implementation also relies on ideal lumped components, which may lead to discrepancies when scaled to high-frequency integrated designs where parasitic effects become significant. Future research will focus on addressing these limitations by extending the method to nonlinear and dynamic systems, incorporating adaptive and machine learning-based real-time tuning algorithms to handle uncertain or rapidly changing load conditions. Further work will also explore the integration of the proposed approach into fully integrated circuit technologies, considering parasitic and manufacturing variations to enhance practical applicability. Finally, experimental validation on a broader range of real-world applications, including wearable and IoT devices, will be pursued to confirm robustness and performance under diverse operational scenarios.

5. CONCLUSION

This study proposed and validated a novel analytical approach for broadband impedance matching based on MAFs, demonstrating its effectiveness through comprehensive theoretical analysis, numerical simulations, and experimental verification on real RF load configurations. The method achieved a 15–25% reduction in maximum reflection coefficient, improved robustness with variations remaining within 2% under ±10% component tolerance, and a 30–40% decrease in optimization iterations, leading to a 20–25% reduction in computational load. Its versatility was confirmed across diverse applications—including multiband receivers, bandpass filters, and antenna matching—highlighting both analytical advantages and practical feasibility. Implementation in a prototype automated design tool further validated its real-world applicability. Overall, the MAF-based technique provides a reliable, computationally efficient, and robust solution for modern broadband matching challenges, with future developments aimed at modeling nonlinear and time-varying behaviors and incorporating machine learning for adaptive real-time tuning.

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AUTHOR CONTRIBUTIONS STATEMENT

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Name of Author	C	M	So	Va	Fo	I	R	D	O	E	Vi	Su	P	Fu
Assel Yerzhan	✓		✓		✓			✓	✓				✓	
Zhanat Manbetova		✓		✓		✓			✓	✓		✓		✓
Gulzada Mussapirova	✓		✓	✓			✓			✓	✓		✓	✓
Gayni Karnakova	✓		✓	✓			✓			✓	✓		✓	✓
Almira Mukhamejanova		✓				✓		✓	✓	✓	✓	✓		
Manat Imankul					✓		✓			✓		✓		✓
Zhanybek Kaliyev		✓				✓		✓	✓	✓	✓	✓		
Nagima Bakirova	✓		✓	✓			✓			✓	✓		✓	✓

C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author, Zhanat Manbetova, upon reasonable request. Due to certain restrictions, including privacy and ethical considerations, the data are not publicly available.

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BIOGRAPHIES OF AUTHORS

Assel Yerzhan in 2003, she graduated from the Almaty Institute of Power and Communications with a degree in industrial electronics. In 2004, she received a Master's degree in Radio Electronics and Telecommunications. In 2013, she graduated from the doctoral program “Kazakh National Technical University named after K.I. Satbayev”, specialty 6D071900 - “Radio engineering, electronics and telecommunications”. From 2014 to the present, she has been a Doctor of Philosophy Ph.D. in the specialty 6D071900 - “Radio Engineering, Electronics and Telecommunications” of the Almaty University of Power Engineering and Telecommunication named Gumarbek Daukeev. She is the author of more than 40 works. Her research interests include circuit engineering in telecommunications, wireless communications, mobile communication systems, GSM and mobile systems management, sensor networks, 5G, and mobile communication technologies. She can be contacted at email: asel.yerzhan@inbox.ru and erjanasel@gmail.com.

Zhanat Manbetova acting Associate Professor of the Radio Engineering, Electronics, and Telecommunications Educational Program Group at the Institute of Energy at NCJSC S. Seifullin Kazakh Agrotechnical Research University. She graduated from Korkyt-Ata State University in Kyzylorda in 1999 with a degree in Physics and Additional Mathematics. In 2014, she graduated from the S. Seifullin Kazakh Agrotechnical University with a master's degree in Radio Engineering, Electronics, and Telecommunications, and in 2022, she defended her doctoral dissertation in the educational program 6D071900 "Radio Engineering, Electronics, and Telecommunications." She has over 25 years of teaching experience and has published more than 50 scientific papers. Her research interests include wireless communication, mobile communication systems, GSM and mobile system management, and mobile communication technologies. She can be contacted at email: zanatmanbetova@gmail.com and zmanbetova@inbox.ru.

Gulzada Mussapirova in 2002, she graduated from the Almaty Institute of Power Engineering and Telecommunications with a degree in industrial electronics in 2006 graduated from Moscow State University named after M.V. Lomonosov. In 2010, she graduated graduate school " Non-profit Joint Stock Company Almaty University of Power Engineering and Telecommunication", specialty 050910 - "Electrotechnologies". From 2011 to the present, she has been Candidate of Technical Sciences in the specialty 050910 - "Electrotechnologies". She is the author of more than 40 works. Currently employed at the Department of Radio Engineering and Telecommunications, Institute "Energy and Digital Technologies", JSC "ALT University named after Mukhametzhan Tynyshpaev". Her research interests include circuit engineering in telecommunications and electrotechnology, wireless communications, mobile communication systems, GSM and mobile systems management, sensor networks, 5G, and mobile communication technologies. She can be contacted at email: g.musapirova@alt.edu.kz.

Gayni Karnakova born in 1980, is a senior lecturer at the Department of Energy of the M.H.Dulati Taraz Regional University. Basic education: M.H.Dulati Taraz State University. Specialty: Automated control and information processing systems, systems engineer. In 2014, she completed her Master's degree at M.H.Dulati Taraz State University. Master's degree in the specialty 6M070200 “automation and control”. In 2023, she completed her doctoral studies at the Lublin University of Technology (Poland) with a degree in automation, electronics, electrical engineering and space technologies. Published more than 50 articles, including 13 articles in journals with a high impact factor according to Scopus web of Science. She can be contacted at email: gaini.karnakova@mail.ru.

Almira Mukhamejanova graduated from the Almaty Institute of Power Engineering and Telecommunications (1998) with a degree in automatic telecommunications. In 2023, she defended her doctoral thesis on the profile 6D071900 – radio engineering, electronics and telecommunications, Ph.D., she is an Associate professor at the Department of Telecommunication Engineering of Almaty University of Power Engineering and Telecommunications named after G.Daukeev, Almaty, Kazakhstan. She is the author of more than 50 scientific papers, including 4 textbooks, 3 articles in the Scopus database. Her research interests include analyses of telecommunication network traffic, and telegraphic theory. She can be contacted at email: a.mukhamejanova@aues.kz.

Manat Imankul in 1979, she graduated from the Tashkent Electrotechnical Institute of Communications with a degree in radio communication and broadcasting. In 1997, she defended her dissertation on the specialty “05.12.13-Radio and television systems and devices” and received the degree of Candidate of Technical Sciences. Currently, she is an associate professor of the Department of Radio Engineering, Electronics, and Telecommunications of the L.N. Gumilyov Eurasian National University. She is the author of more than 200 scientific papers, including 1 monograph, 2 articles in the Scopus database. Research interests-wireless communication technologies. She can be contacted at email: mimankul57@gmail.com.

Zhanybek Kaliyev Doctor Ph.D., assistant professor of the Department “Energy” of the Institute “Automation and Telecommunications”, JSC Academy of Logistics and Transport. In 2011, he defended his Ph.D. thesis on the theme “Integrated software package for the analysis of algorithms for managing the subway train” in the specialty 05.13.06 - “Automation and control of technological processes and industries.” Since 2014 Ph.D. in specialty 6D070200 - Automation and control. His research interests include energy-optimal control of metro train traffic, mathematical modeling, electric power industry, and renewable energy sources. He can be contacted at email: zhanibek.84@mail.ru and zh.kaliyev@alt.edu.kz.

Nagima Bakirova in 2003, she graduated from the Almaty Institute of Power Engineering and Telecommunications with a degree in industrial electronics. In 2004, he received a Master's degree in Radio Electronics and Telecommunications. From 2004 to the present time, she has been working at the Department of Department of Radio Engineering and Telecommunications, Institute "Energy and Digital Technologies", JSC "ALT University named after Mukhametzhan Tynyshpaev. In 2024, she entered the doctoral program Academy jf Logistics and Transport in the specialty 8D06255 - radio engineering, electronics and telecommunications. She is the author of more than 20 works. Her research interests include circuit engineering in telecommunications, wireless communications, mobile communication systems, GSM and mobile systems management, sensor networks, 5G, and mobile communication technologies. She can be contacted at email: bakirovanagima812@gmail.com.