

## Frequency dependency analysis for differential capacitive sensor

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### ABSTRACT

A differential capacitive sensing technique is discussed in this paper. The differential capacitive sensing circuit is making use of a single power supply. The design focus for this paper is on the excitation frequency dependency analysis to the circuit. Theory of the differential capacitive sensor under test is discussed and derivation is elaborated. Simulation results are shown and discussed. Next, results improvement has also been shown in this paper for comparison. Test was carried out using frequency from 40 kHz up to 400 kHz. Results have shown output voltage of  $V_{out}=0.07927 C_x+1.25205$  and good linearity of R-squared value 0.99957 at 200 kHz. Potential application for this capacitive sensor is to be used for energy harvesting for its potential power supply.

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## 1. INTRODUCTION

In measurement system, when two conductive materials are arranged in parallel and separated by non-conductive or dielectric element, a charge is stored in the capacitor in terms of electric fields. A typical dielectric material is air, plastic or ceramic and recently, organic solvent has been widely used for Lab-on-Chips, LoC purpose [1]. A common capacitance range used is between 1 pF and 1000  $\mu$ F. Depending on the design and method used in the system, capacitance is first converted into voltages and continue its conversion process into desired forms such as frequency, duty cycle, digital and phase.

In CMOS realization for very high speed [2], a switched-capacitor method is best suited for this application, in addition to its immune to stray capacitances [3]. However, it results to moderate resolution due to incapable of handling capacitance changes with frequency higher than 10 Hz [4]. In Capacitance-to-frequency design, higher resolution is achievable [5]. However, it suffers nonlinearity problem due to poor accuracy at high frequency range [6].

On the other hand, in charge/discharge method, the output voltage is depending on the sampling time [3]. The sampling time of this charge/discharge converter are limited by the measured capacitance value. The larger the capacitance, the larger the charge and discharged time [3]. Other method that used double sensing elements is called differential capacitive sensor method. This method is effective to overcome the previous problems mentioned earlier such as it can cancel parallel stray capacitance at sensor connected cable [7] and it can provide good accuracy even at high frequency of MHz operation [8].

This paper is about differential capacitive sensor that uses discrete components and has utilized single supply to source to its elements like oscillator, operational amplifiers, voltage divider circuit and instrumentation amplifier. The focus of this paper is to analyse the effect of frequency dependency to the

differential capacitive circuit. It is the addition study on work done in [9]. The frequency under test is at 200 kHz. Based on the study, the proposed circuit can also work under frequency from 40 kHz up to 400 kHz.

## 2. FREQUENCY DEPENDENCY TRANSFER FUNCTION

In a very low differential capacitance sensing circuit where the output voltage of the amplifier is almost at 0 V voltage level, a proper output voltage could not be obtained when  $\Delta C_x$  is less than the nominal  $C_{x0}$ . This is because it has exceeded the negative side. There is a need to set some reference level higher than 0 V voltage level at positive side, so that both positive and negative output voltage amplitude of  $V_{out'}$  could be observed only at positive side. This can be done by giving an input DC voltage, i.e. the reference voltage,  $V_{ref}$  to the non-inverting input of the Opamp that acts as a voltage level shifter is shown in Figure 1. This Figure 1 is an extracted version of Figure 2 in [9, 10]. At one side of the differential capacitive sensing, the output voltage of the operational amplifier is:

$$V_{out'} = \left[ 1 + sC_x \frac{R_f}{1 + sR_f C_f} \right] V_{ref} - \left[ sC_x \frac{R_f}{1 + sR_f C_f} \right] V_{ext} \quad (1)$$

where the  $s$  is corresponding to  $j\omega$

Several conditions may exist when the reference voltage,  $V_{ref}$ , is supplied to the non-inverting input of the amplifier. Based on (1), the output voltage is divided into several conditions as the following:

### Case I: $V_{ref}=0$

$$\left| \frac{V_{out'}}{V_{ext}} \right| = -sC_x \frac{R_f}{1 + sR_f C_f} \quad (2)$$

### Case II: $V_{ref}=V_{ext}$

$$\left| \frac{V_{out'}}{V_{ext}} \right| = 1 \quad (3)$$

### Case III: $V_{ref} < V_{ext}$ when $V_{ref}=(1-m)V_{ext}$ where $(0 < m < 1)$

$$\left| \frac{V_{out'}}{V_{ext}} \right| = 1 - \left( 1 + sC_x \frac{R_f}{1 + sR_f C_f} \right) m \quad (4)$$

### Case IV: $V_{ref} > V_{ext}$ when $V_{ref}=nV_{ext}$ where $(n > 1)$

$$\left| \frac{V_{out'}}{V_{ext}} \right| = n \left( 1 + sC_x \frac{R_f}{1 + sR_f C_f} \right) - sC_x \frac{R_f}{1 + sR_f C_f} \quad (5)$$

where  $m$  and  $n$  are the excitation input voltage,  $V_{ext}$  multiplier numbers which is equal to  $nV_{ext}$  or  $(1-m)V_{ext}$

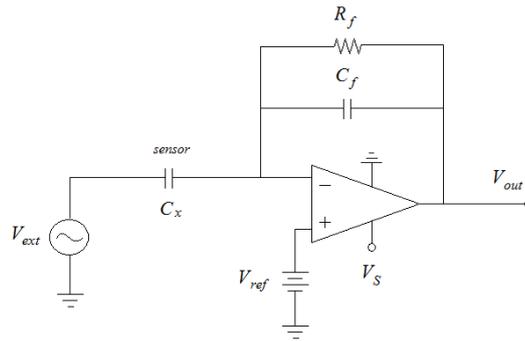


Figure 1. Vref at non-inverting differential amplifier input as level shifter

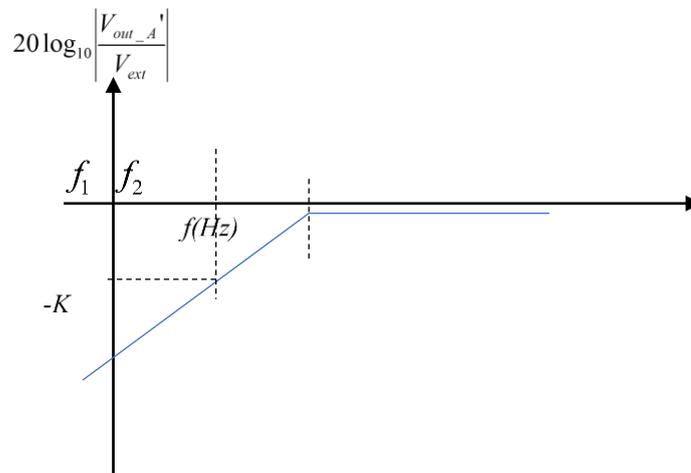


Figure 2. Estimated bode plot for transfer function at opamp a output

Referring to the Figure 2 of [9, 10], the design begins by deriving the transfer function *opamp A* output to the input,  $V_{ext}$ , by setting the range of frequency to pass the signal. A continuous supply of 3.3 V is provided to the circuit and voltage divider is used to level the output voltage to half of supply voltage, which in this case is 1.65 V. The reason is to set the signal in positive range. An excitation supply voltage in terms of sinusoidal waveform,  $V_{ext} \cos(2\pi f_{ext}t + \varphi)$  is used. The test starts with analysis without parasitic impedance to the differential Capacitance-to-Voltage Converter (CVC) circuit. Without the parasitic impedance, the transfer function derivation of the cutoff frequency amplifiers output,  $V_{out\_A'}$  (refer to Figure 2 of [9, 10]) at *opamp A* output is:

$$\frac{V_{out\_A'}}{V_{ext}} = -\frac{s R_f C_x}{1 + s R_f C_f} = -\frac{R_f C_x}{R_f C_f} \frac{s}{1 + \frac{s}{(R_f C_f)^{-1}}} = K f_1 \frac{s}{1 + f_2 s}$$

where

$$K = -\frac{C_x}{C_f} \tag{6}$$

$$f_1 = 1$$

$$f_2 = \frac{1}{R_f C_f}$$

Note that the apostrophe (') sign is a punctuation mark to signal the condition without parasitic capacitance and resistance present in the circuit whereas Figure 2 of [9, 10] does not showing this sign. Estimated bode plot theory of *opamp A* output is sketched as in Figure 2. The straight line is pictured as in

Figure 1 if only if the corner frequency  $f_2$  is small and near to frequency  $f_1$ , else the magnitude is at positive side and the 20 dB/dec line will crosses 0 dB. The extended derivation of [11] for both amplifiers output transfer function is as in the following:

$$V_{out\_A}' = - \left( \frac{(C_x + \Delta C)}{C_f} \cos 2\pi f_{sensor} t \right) V_{ext} \cos 2\pi f_{ext} t \quad (7)$$

$$V_{out\_B}' = - \left( \frac{C_r}{C_f} \right) V_{ext} \cos 2\pi f_{ext} t \quad (8)$$

Accordingly, the transfer function taken at  $V_{filterA}'$  and  $V_{filterB}'$  [9, 10] using the first case (Case I:  $V_{ref}=0$ ) of (2).  $C_r$  is a reference capacitor with value equal to nominal value  $C_{x0}$

$$V_{out}' = G \left[ \left( \frac{C_r}{C_f} - \frac{(C_x + \Delta C)}{C_f} \cos 2\pi f_{sensor} t \right) \hat{V}_{ext} - V_{diode\_A}' + V_{diode\_B}' \right] \quad (9)$$

When  $V_S=3.3$  V and  $V_{ext}=400$  mV,  $V_{ref}$  will be half of  $V_S$ , and it is calculated that the  $n$  to be equal to 4.125. For (Case IV:  $V_{ref} > V_{ext}$ ) when  $V_{ref}=4.125V_{ext}$  from (5) it is derived that the output is as in (10)-(11).

$$\left| \frac{V_{out}'}{V_{ext}} \right| = 4.125 \left( 1 + sC_x \frac{R_f}{1 + sR_f C_f} \right) - sC_x \frac{R_f}{1 + sR_f C_f} \quad (10)$$

And

$$\frac{V_{out\_A}(V_{ref} > V_{ext})'}{V_{ext}} = 4.125 + 3.125 \frac{R_f C_x}{R_f C_f} \frac{s}{1 + \frac{s}{(R_f C_f)^{-1}}}$$

where

$$K = \frac{C_x}{C_f} \quad (11)$$

$$f_1 = 1$$

$$f_2 = \frac{1}{R_f C_f}$$

$\hat{V}_{ext}$  is the excitation amplitude voltage after demodulation (i.e.  $\hat{V}_{ext}=400$  mV). Since the same Schottky diode is used for the circuit, the different between both diode voltage is only slightly different caused by changes in  $C_x$  (i.e: the difference is  $\pm 0.1$  pF), for simplification both as  $V_{diodeA}'=V_{diodeB}'$  is set equal. The final output voltage is:

$$V_{out(V_{ref} > V_{ext})}' = G \left[ 3.125 \hat{V}_{ext} \left[ \frac{(C_x + \Delta C)}{C_f} \cos 2\pi f_{sensor} t - \frac{C_r}{C_f} \right] \right] \quad (12)$$

The relevance of this equation is, when there is no capacitance change in  $\Delta C_x$ , there is no movement at the sensor so,  $f_{sensor}=0$  and when both  $C_r$  and  $C_x$  at nominal capacitance value  $C_{x0}=5$  pF, the resulted output voltage  $V_{out}=0$ . The gain  $G$  need to be within the limit and not exceeding the  $+V_{DD}$  for the relevance output value as in [12, 13].

### 3. OPAMP TRANSFER FUNCTION DEPENDENCY TO THE EXCITATION FREQUENCY RESULTS AND ANALYSIS

In this test, the frequency dependency is investigated by varying the frequency from 1 kHz to 10 GHz, with voltage amplitude of 400 mV is kept constant. The resulted bode plot is verified by checking the dependency of excitation frequency to the derived transfer function. For (2) of Case I:  $V_{ref}=0$ , using  $C_f=C_x=C_r=C_{x0}=5 \text{ pF}$  and  $R_f=10 \text{ M}\Omega$ . Simulation result of Figure 3 has shown almost to the estimated theoretical value (6) and expected corner frequency  $B$ , is 3.183 kHz.

Figure 4(a) shows the effect of changing in reference capacitance,  $C_f$  to the magnitude of operation across frequencies at the final output. The feedback resistor,  $R_f$ , at this point is set to 10 MΩ. Changing of  $C_f$  value has caused magnitude shift to the passband frequencies. While changing the value of  $R_f$  has affected to magnitude shifting in the final output of the whole circuit as shown in Figure 4(b). Effect of changing in reference resistance,  $R_f$  at the feedback loop to the frequencies of operation when  $C_f$  is set to 5 pF.

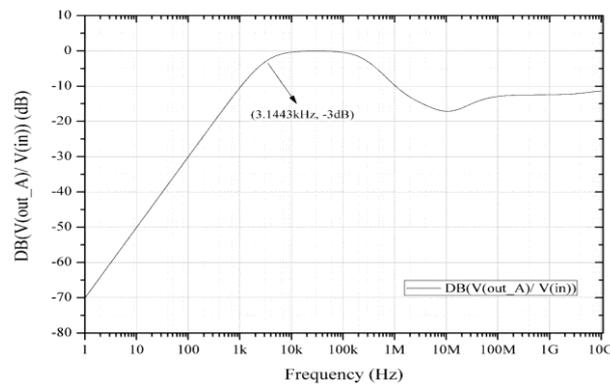


Figure 3. Simulated magnitude result at opamp a output

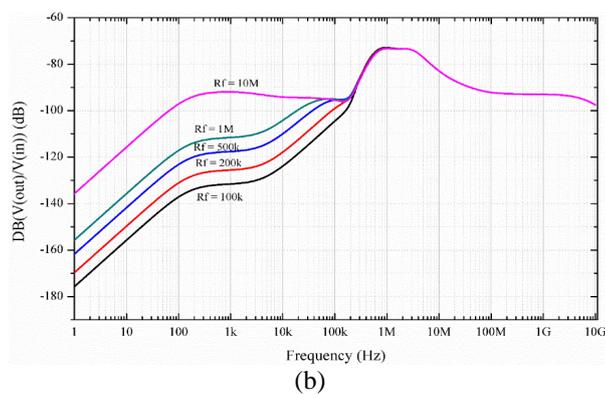
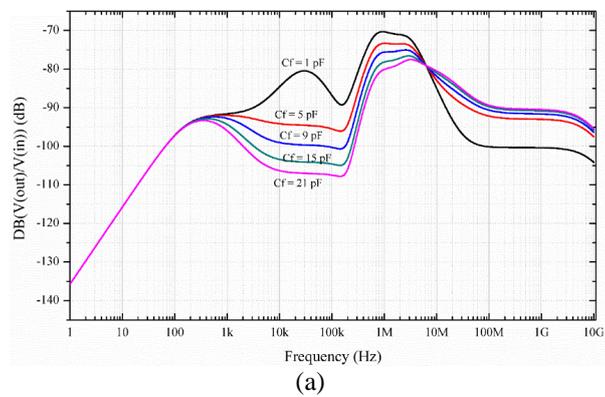


Figure 4. Effect of changing, (a) reference capacitance,  $C_f$  (b) reference resistance,  $R_f$  to the magnitude across frequencies

### 3.1. Before improvement of capacitance detection change, $\Delta C_x$

The overall output voltage linearity graph is plotted in Figure 5 when frequency is at 200 kHz and simulation is using  $R_f=10\text{ M}\Omega$ ,  $R_d=100\text{ k}\Omega$  and  $C_d=0.1\text{ }\mu\text{F}$ . The 200 kHz frequency was chosen to test the possibilities and condition of the differential amplifier when running at frequency higher than 100 kHz. When frequency goes higher than 100 kHz, which is shown in Figure 5, less points of detection were observed and the sensitivity is low. By rules, the  $f_{-3dB}$  should be low than the cutoff frequency ( $f_c=3.18\text{ kHz}$ , refer to the bode plot of Figure 2). In this case has satisfied the condition when using  $R_d=100\text{ k}\Omega$  and  $C_d=0.1\text{ }\mu\text{F}$ , the  $f_{-3dB}=15.9\text{ Hz}$ .

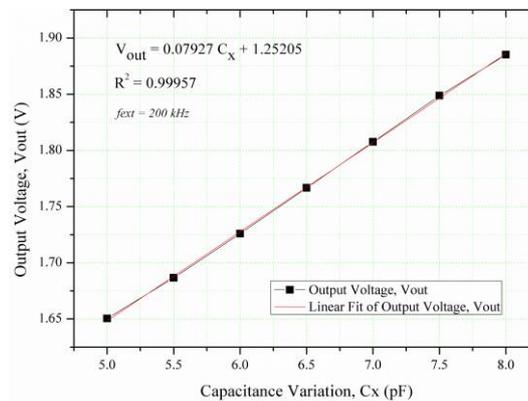


Figure 5. Effect of output voltage with  $C_x$  variation for  $R_d=100\text{ k}\Omega$  and  $C_d=0.1\text{ }\mu\text{F}$  using  $V_s=3.3\text{ V}$  at 200 kHz

Improvement on the number of detection points across certain capacitance range is increased by increasing the bandwidth of the low pass filter. This is by decreasing the  $R_d$  value and  $C_d$  value of the components. Figure 6 shows the simulation results using low  $R_d$  and  $C_d$  values of 10 k $\Omega$  and 0.01  $\mu\text{F}$  respectively, with  $R_f=10\text{ M}\Omega$ . In this case, the cutoff frequency,  $f_{-3dB}=1.59\text{ kHz}$ , which satisfy the condition less than cutoff frequency  $f_c=3.183\text{ kHz}$ .

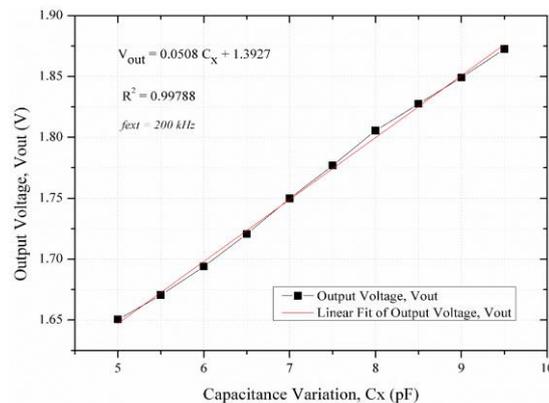


Figure 6. Effect of output voltage with  $C_x$  variation for  $R_d=10\text{ k}\Omega$  and  $C_d=0.01\text{ }\mu\text{F}$  using  $V_s=3.3\text{ V}$  at 200 kHz

### 3.2. After improvement of capacitance detection change, $\Delta C_x$

Improvement is made to the capacitance change,  $\Delta C_x$  so that wider capacitance range is been detected with low detection change at high frequency. This is done by using the  $R_f$  selection method. This method is considered relevant due to diode current value at higher frequency (i.e: 200 kHz) is becoming stable regardless of the change of the  $R_f$  value (refer to diode current of Figure 4. 10 for  $R_d=10\text{ k}\Omega$  and  $C_d=0.01\text{ }\mu\text{F}$ ). Same principle applied to any components value at higher frequency, such as when  $R_d=10\text{ k}\Omega$  and  $C_d=0.1\text{ }\mu\text{F}$  of Figure 2 of [14].

Figure 7 shows the linearity result using  $\Delta C_x=0.1$  pF change with  $R_d=10$  k $\Omega$  and  $C_d=0.1$   $\mu$ F. Different  $R_f$  has been selected to overcome the sensitivity problems where Figure 7 is at  $R_f=10$  M $\Omega$  at  $f_{ext}=200$  kHz. These values must satisfy the condition  $>f_{.3dB}$  range. In this case the frequencies are 3.183 kHz, 159 kHz and 106 kHz respectively, when  $f_{.3dB}=159.13$  Hz. At high frequency, high sensitivity of capacitive change of detection is achieved by reducing the value of the feedback resistor  $R_f$  and other results of different analysis can also be found in [15].

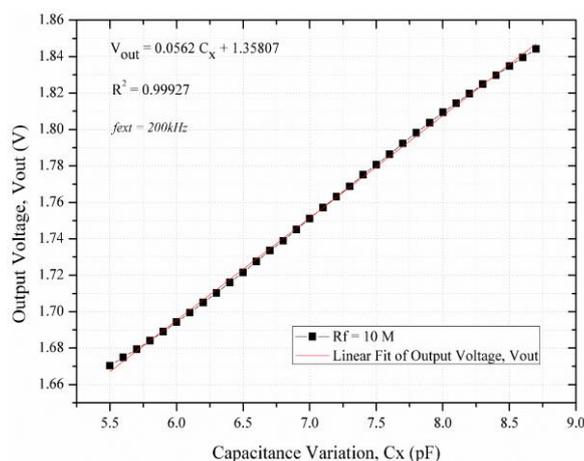


Figure 7. Corrected capacitance change,  $\Delta C_x=0.1$  pF with  $R_f=10$  M $\Omega$  at  $f_{ext}=200$  kHz

#### 4. CONCLUSION

In summary, this paper presented a differential capacitive sensing with output voltage of  $V_{out}=0.07927 C_x+1.25205$  with R-squared value of 0.99957 at frequency 200 kHz. Using nominal capacitance of 5 pF and  $R_f=10$  M $\Omega$ , simulation result has shown almost to the estimated theoretical value and expected corner frequency of 3.183 kHz. The change of  $C_f$  value has caused magnitude shift to the passband frequencies. While changing the value of  $R_f$  has affected to magnitude shifting in the final output of the whole circuit. In future, a proposed solution can be done to improve points of detection of output voltage capacitance sensing across frequencies, by properly setting the values of component involved such as the resistance and capacitance of the differential capacitive sensing.

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