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Aspects in Formulating Mathematical Model of Wind Turbine

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Abstract

The present paper explores the mathematical molding of the wind turbine and its influence on the subsequent stages. Specifically, the paper investigate the modeling of gear train of the wind turbine and distinguishes the difference in the approaches usually used to establish the mathematical model which is later has a significant impact on the design, characteristic and performance of the modeled system. Mainly two commonly used approached for the gear train systems are analyzed and discussed. The main well know mechanisms are investigated in term of the most proposed assumptions to deal with the damping, shaft stiffness and inertia effect of the gear. This paper elucidates these concerns.

Keywords: *mathematical, model, turbine, wind.*

1. Introduction

It is well known that the renewable energy industry is accelerating quickly and intensively rising concerns about natural recourses exhaustion and climate change. Renewable energy is seen by many as part of the appropriate response to these concerns and some national Governments have put programs in place to support the wider use of sustainable energy systems [1]. As a consequence, this led to a rapid boost in demand for renewable energy specialists who should be able to design, install and maintain such systems. Most engineers are not trained to use these renewable energy technologies and most are not aware of the principles of sustainability. There is therefore an urgent need to develop and implement new university courses that prepare engineers, scientists and energy planners to work with renewable to produce sustainable energy generation systems. However, this rapid growth has worsened the problem of a serious shortage of experienced professionals in renewable energy. The type of person in demand includes designers, installers, service and sales representatives, policy analysts, academic scientists, engineers, teachers and researchers. Without them the quality of systems may be compromised and the demand for renewable energy may be adversely affected as a result [2].

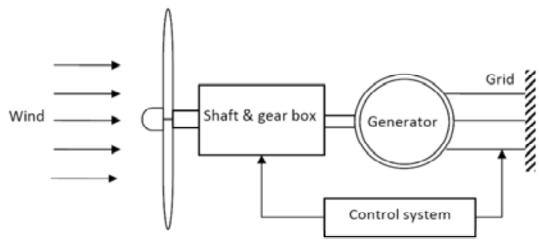

Figure 1. Components of a Typical Wind System

Wind turbines produce electricity by using the power of the wind to drive an electrical generator. Wind passes over the blades, generating lift and exerting a turning force. The rotating blades turn a shaft inside the nacelle, which goes into a gearbox. The gearbox increases the rotational speed to that which is appropriate for the generator, which uses

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magnetic fields to convert the rotational energy into electrical energy. The power output goes to a transformer, which converts the electricity from the generator to the appropriate voltage for the power collection system [3]. The basic components involved in the representation of a typical wind turbine generator are shown in Figure 1.

An overall wind energy system can be divided into following components [4]:

a) Model of the wind,
b) Turbine model,
c) Shaft and gearbox model,
d) Generator model and
e) Control system model.

The mechanical parts of the wind turbine generator are the first three components. Basically, the generator produces the electro-mechanical link between the turbine and the power system and the control system controls the output of the generator. In general, the concept of using the wind energy to obtain the electricity power can be represented by the following diagram:

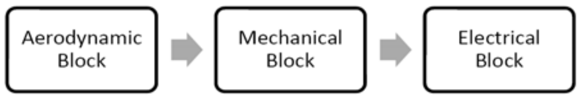


Figure 2. Wind Turbine Block Diagram

The mechanical system of the wind turbines plays a big role in the energy transformation. Most of the simple wind turbine gear box consists of two main shafts, the low speed shaft which is basically connected with the wind turbine blades, and the second one which is called the high speed shaft connected directly to the generator and shown in Figure 3.

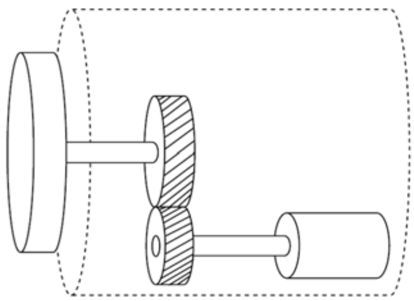
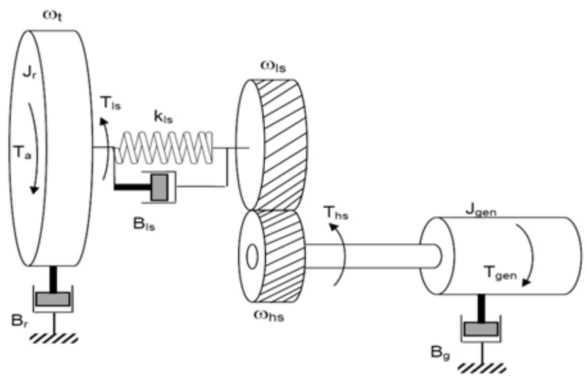


Figure 3. Typical Gear System of Wind Turbine

The present paper simply investigates the differences in the mathematical modeling approaches that commonly using to model the actual adopted mechanical system. Mainly two techniques are explained and detailed with the assumptions used in order to distinguish the differences and hence the consequences of the modeling.

2. Case I

This case of the two mass models is considered the most commonly used to describe a two-mass system [5-7]. Mainly, to describe the wind turbine system which is basically includes the blade, gear train and the generator which is shown in Figure 4. As a matter of fact, this model is adopted for further modeling for the turbine control system mathematical model which is established based on this model, and mainly for wind turbine of different sizes.



In particular, most controllers are considered more adapted for high-flexibility wind turbines that cannot be properly modeled with a one mass model [8]. Besides, it is shown that [9] the two-mass model can show flexible modes in the gear train model that cannot be done using the one mass model. Besides, this model was also used to investigate the linear and the nonlinear controller of the wind turbine [10]. The essential assumptions of the present model are:

a) Considering the blade damping effect and the generator damping effect as well.
b) Eliminating the impact of the high speed shaft stiffness.
c) Cancellation the driving and the driven gears and the shafts moment of inertias.

By applying Newton's second law for rotation system or using energy principles on the rotor, the mathematical model will be:

$$J_r\dot{w}_t + B_r w_t = T_a - T_{ls} \quad (1)$$

Where:
J_r =rotor moment of inertia
ω_t =roto angular speed
B_r=rotor damping effect
T_a=applied torque on the rotor
T_{ls}=low speed shaft torque
Whereas the same technique is applied for the driving gear, which basically its moment of inertia is cancelled, this will yield:

$$J_{ls}\dot{w}_{ls} + B_{ls}(w_t - w_{ls}) + K_{ls}(\theta_t - \theta_{ls}) = T_{ls} \quad (2)$$

Where:
J_{ls}=driver moment of inertia (cancelled)
ω_{ls} = angular speed of the low speed shaft
B_{ls}=low speed damping effect
K_{ls}=stiffness of low speed shaft
θ_t =rotor angular displacement
θ_{ls} =low speed angular displacement
This will yield:

$$T_{ls} = B_{ls}(w_t - w_{ls}) + K_{ls}(\theta_t - \theta_{ls}) \quad (3)$$

By the same procedure, the mathematical model for the generator is:

$$J_g\dot{w}_g + B_g w_g = T_{hs} - T_{em} \quad (4)$$

Where:
J_{ls}=generator moment of inertia

ω_g = angular speed of the high speed shaft
B_g=high speed damping effect
T_{hs}=high speed shaft torque
T_{gen}=generator electromagnetic torque
The gear train ratio n_g is described by:

$$n_g = \frac{T_{ls}}{T_{hs}} = \frac{w_g}{w_{ls}} = \frac{\theta_g}{\theta_{ls}} \quad (5)$$

Whereθ_g = angular displacement of high speed shaft.
By differentiating Equation (3) with respect to t and organizing Equation (1) and (4) in term of ω_t and ω_g respectively and using Equation 5, the matrix form of the variables can be expressed:

$$\begin{bmatrix} \dot{w}_t \\ \dot{w}_g \\ \dot{T}_{ls} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{21} & a_{31} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} w_t \\ w_g \\ T_{ls} \end{bmatrix} + \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix} T_a + \begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix} T_{em} \quad (6)$$

$$a_{11} = \frac{-B_r}{J_r}, a_{12} = 0, a_{13} = \frac{-1}{J_r}, a_{21} = 0, a_{22} = \frac{-B_g}{J_g}$$

$$a_{23} = \frac{1}{n_g J_g}, a_{31} = \left(K_{ls-} \frac{B_{ls} B_r}{J_r}\right), a_{32} = \frac{1}{n_g}\left(\frac{B_{ls} B_r}{J_g} - K_{ls}\right)$$

$$a_{33} = -B_{ls}\left(\frac{J_r + {n_g}^2 J_g}{{n_g}^2 J_g J_r}\right), b_{11} = \frac{1}{J_r}, b_{12} = 0, b_{21} = 0$$

$$b_{22} = \frac{-1}{J_g}, b_{31} = \frac{B_{ls}}{J_r}, b_{22} = \frac{B_{ls}}{n_g J_g}$$

The present mathematical model of the variable can be used for the subsequent stages for the derivation of the controller model [10].

3. Case II

The second approach to find the mathematical model of the mechanical system is based on the idea of establishing the comprehensive mathematical model of the mechanical system by including mainly the effect of the moment of inertias of the gears besides the stiffness of the high speed shaft which was cancelled in the previous technique. The present procedure is called the three-mass model. Accordingly, the developed mathematical model is used to find the equivalent two-mass model as well as the one-mass model.

3.1. Three Masses Model

The diagram of the three-mass model is shown in Figure 5.

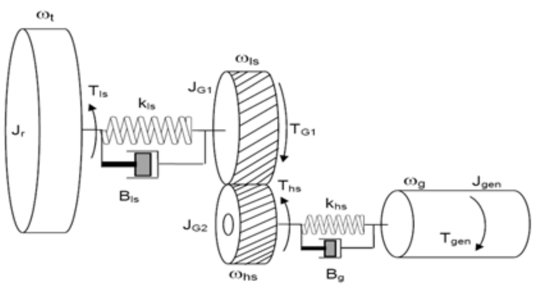


Figure 5. Case II Gear System: Three Masses

By using either the energy method or the Newton's second law for rotational systems, the first mathematical model of the wind turbine rotor can be written as following [11]:

$$T_{ls} = J_r \dot{w}_t + B_{ls} w_t + k_{sl}(\theta_t - \theta_{ls}) \quad (7)$$

Here it is important to mention that the effect of the damping on the low speed shaft is only considered on the rotor side one time and the gear one on the second time, whereas in case I, this damping effect was considered as a relative between them.

Since the moment of inertia of gear one is considered, and then the mathematical model of it is calculated as:

$$T_{G1} = J_{G1} \dot{w}_{ls} + B_{ls} w_{ls} + k_{sl}(\theta_{ls} - \theta_t) \quad (8)$$

Whereas for gear 2, the dynamic equation is written as:

$$T_{hs} = J_{G2} \dot{w}_{hs} + B_g w_{ls} + k_{hs}(\theta_{hs} - \theta_g) \quad (9)$$

Again here, the damping effect of the generator B_g is not considered as relative between gear 2 and the generator.

Finally, for the generator, the differential equation is represented by:

$$-T_{gen} = J_{gen} \dot{w}_g + B_g w_g + k_{hs}(\theta_g - \theta_{hs}) \quad (10)$$

Where T_{G1} is the torque on gear one, J_{G1} and J_{G2} are the moment of inertial for both gear one and two respectively.

3.2. Two Masses Model

Once the tree-mass mathematical model is obtained, the system can be simplified to be two-mass model through reforming the dynamic system with the equivalent stiffness and damping coefficients, k and B respectively. In this context, and in order to utilize the previous mathematical model developed, it is necessary to assume that the moment of inertia of the shafts and the gears are neglected because their impact on the behavior of the system is relatively tiny in comparison with the moment of inertia of the wind turbine or even generator. As a consequence, the proposed two-mass model is consisting of two main rotors, which represent the wind turbine and the generator and connected by a flexible shaft, so the equivalent developed system is shown in Figure 6.

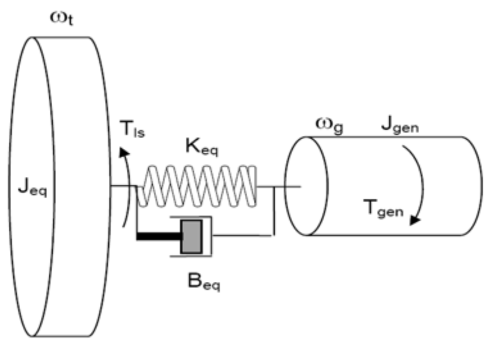


Figure 6. Case II Gear System: Two Masses

In order to build the mathematical model of the new two-mass system, it is important to mention that the influence of the gear ratio must be included, which is represented by:

$$K_{gear} = \frac{w_{hs}}{w_{ls}} = \frac{T_{G1}}{T_{hs}} \quad (11)$$

Where K_{gear} is the gearbox ratio.
Therefore, the dynamic system can be formulated with respect to the wind turbine rotor one time and the electromagnetic generator in the second time which is connected by the gearbox. For the wind turbine rotor, the mathematical model is written as:

$$T_{ls} = J_{eq2}\dot{w}_t + B_{eq}(w_{ls-}w_g) + k_{eq2}(\theta_t - \theta_g) \quad (12)$$

For the wind turbine side, whereas for the generator side, the mathematical model is:

$$-T_g = J_{gen}\dot{w}_g + B_{eq}(w_{g-}w_{ls}) + k_{eq2}(\theta_g - \theta_t) \quad (13)$$

Where the equivalent stiffness if specified by:

$$\frac{1}{k_{eq2}} = \frac{1}{\frac{k_{ls}}{k_{gear}{}^2}} + \frac{1}{k_{gen}} \quad (14)$$

The equivalent moment of inertia is given by:

$$J_{eq2} = \frac{J_r}{k_{gear}{}^2} \quad (15)$$

3.3. One Mass Model

Eventually, whenever the shat stiffness and the damping effect are ignored additional reduction of the mathematical model can be obtained. The one-mass model is acquired in this situation, which is described by:

$$J_{eq1}\dot{w}_g = T_{gen} - T_{eq1} \quad (16)$$

Where the equivalent moment of inertia and the torque is represented by the following equations respectively:

$$J_{eq1} = J_{gen} + \frac{J_r}{k_{gear}{}^2}$$

$$T_{eq1} = \frac{T_{ls}}{k_{gear}{}^2}$$

4. Conclusion

The present paper shows the differences between two commonly used methodologies which are mainly adopted by the academics, researchers and engineers. Mainly, the differences between the approaches in the modeling are the damping effect of the wind turbine blades besides the impact of the shaft's stiffness. Knowing the details of these differences and the similarities becomes as a helpful guidelines and it makes things easily to those who intend to develop or even start researches and even can be used for teaching in undergraduate and graduate courses as a principle of wind turbine modeling.

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