Tracking controller for uncertain wheel mobile robot: adaptive sliding mode control approach

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ABSTRACT

This paper presents a technique for developing a sliding mode controller (SMC) using the state model of wheel mobile robot (WMR). The control scheme which consists of a controller and a disturbances observer can eliminate system uncertainties, disturbances, and unknown wheel slips. To successfully implement the sliding mode tracking controller algorithm, first, the transformation is utilized to convert the kinematic and dynamic models to an equation of state, and then create a controller based on Lyapunov function. Subsequently, a disturbance observer is formulated based on a stable sliding surface, followed by the development of an adaptive sliding mode control (ASMC) for the system. Moreover, to verify the efficacy of the given strategy, simulations have been performed under the aforementioned disturbance conditions. Finally, the simulation results show that chattering effect is eliminated, and the impact of disturbances is also diminished, thus proving the viability and correctness of the proposed control algorithm.

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1. INTRODUCTION

Wheel mobile robot (WMR) are simple in structure, certainty, and high energy efficiency [1]. The WMR have many applications in various fields such as transportation, military, medical, and rescue. Underactuated WMR which is popular among mobile robots, is a non-linear subject with multi-input, multiple-output, and has nonholonomic constraints. As a result, this WMR has attracted the interest of many researchers working on modeling and control algorithms [1]–[3].

In recent decades, many literatures have been published with a variety of control methods for tracking control of WMR. Among these, the sliding mode control (SMC) [4]–[10] is one of the most popular technique which is employed for WMR to cancel or reduce the effect of unknown components to the working of the system. The control schemes in [4], [5] are traditional SMC for closed-loop dynamics. The cascade control structure provides a good control quality and becomes a favorable structure for several current researches. Also, cascade control structure is introduced in [6] for tracking control of WMR. The kinematic controller is designed based on Lyapunov function and the dynamic control loop is stabilized by SMC. The presented control algorithm guarantees the fast speed tracking and eliminates the kinematic uncertainties. According to Goswami and Padhy [7], the SMC is employed for both inner-loop and outer-loop to cancel the effect of disturbances. In this work, the wheel slips are integrated into the system model and this becomes popular in later works. According to Zhai and Song [8], the nonsingular terminal SMC without any constraints is developed for WMR with external disturbances and inertia uncertainties considering. The proposed control scheme ensures the finite-time convergence but the tracking error only converges to a small region. Although all of the above SMCs provide good performances for the tracking control problems of the WMR, a common drawback of SMC, chattering phenomena, still exists. For this reason, many researchers are constantly studying and proposing algorithms to improve the performances of the SMC systems.

Parallel with the SMC, adaptive control techniques [11]–[25] are also employed to solve the problems of the WMR systems. According to Amer *et al*. [13], the backstepping control is used for kinematic loop and fuzzy SMC for dynamic loop to achieve the robustness and the stability. The fuzzy logic system (FLS) is utilized to tune the adaptation gain which increases the adaptability and chattering free. However, despite the reasonability of the theory, the demonstrated result in this work is unclear. Another approach of the adaptive control scheme which employed the neural network (NN) controller based on the reinforcement learning method is proposed in [20]. In this work, NNs are trained to address the uncertainties, the slipping and the skidding components which affect the tracking quality of the system. With this structure, the system performances are improved countably; however, the complexity is one of the disadvantages of this scheme. Designing an adaptive control in combination with a disturbance observer is also one of the most effective ways to compensate the effect of disturbance [22], [23]. Especially, according to Li *et al*. [22], the cascade structure which incorporates a nonlinear disturbance observer with the extended Kalman filter is used to avoid the influence of disturbance. Although the wheel slipping can be compensated, the overshoot of the performance is still too high. Meanwhile, the strategy in [23] is also the two-loop structure with a disturbance observer in the second loop. With the addition of the observer, the feedforward compensation for the lumped disturbance is obtained, but the settling time in the result is large. Besides, to improve the control quality, [24] also proposes a control scheme which combines the sliding-mode control and disturbance observer. In this case, two observers are executed to eliminate the effect of the disturbance. However, this structure increases the number of complex calculations tremendously.

This paper introduces an adaptive SMC for the purpose of tracking control of a WMR in the presence of uncertainties, input disturbances, and wheel slips. The establishment of the state space model of the system is primarily based on the kinematic and dynamic models. Subsequently, the state space model is stabilized through the implementation of the adaptive SMC scheme. The unidentified elements within the system are approximated through the utilization of an observer, which subsequently transmits the information to the controller in order to adjust for the impact of any disruptions. The observer's convergence is guaranteed through the utilization of Hurwitz criteria, while the stability of the entire system is established on the basis of the Lyapunov theory. The advantages of the proposed scheme are stated as: i) the control system is designed with only one loop instead of two loops as usual. This structure reduces the complexity in both study and implementation and ii) the control signal is free from chattering phenomena of the SMC which gives the reduction of the cost of energy, extending the acuator’s lifespan. Finally, the comparative simulations are done to demonstrate the advantages of the proposed adaptive SMC in comparison with the existing work.

2. MATHEMATICAL MODEL

In the article, the study focuses on the WMR concerning the robot's motion on a horizontal surface. The body of the system incorporates two rear-wheel drive wheels for propulsion and a front steering wheel. The movement frames of the WMR are illustrated in Figure 1.

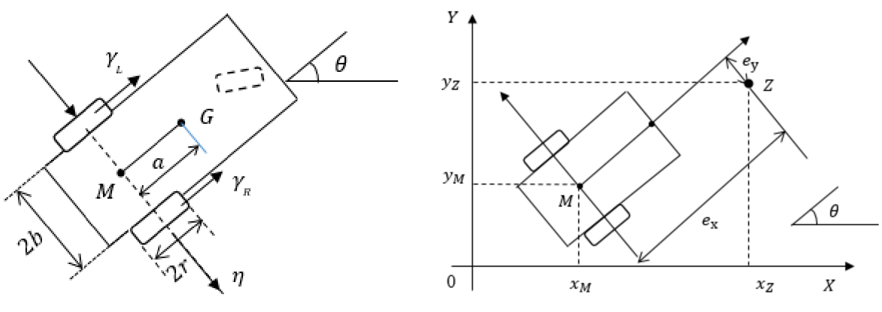


Figure 1. Wheel mobile robot model

Let γ_R, γ_L are the longitudinal slip factor of the left and right driving wheels, η is the lateral slip factor along the wheel shaft. Angular velocity of the right and left wheel are $\dot{\phi}_R$, $\dot{\phi}_L$ respectively. Linear forward velocity in perpendicular direction of the wheel shaft and angular velocity are calculated as (1) [12]:

$$\vartheta = \frac{r(\dot{\phi}_R+\dot{\phi}_L)}{2} + \frac{\dot{\gamma}_R+\dot{\gamma}_L}{2}, \quad \omega = \frac{r(\dot{\phi}_R-\dot{\phi}_L)}{2b} + \frac{\dot{\gamma}_R-\dot{\gamma}_L}{2b} \tag{1}$$

The mathematical model of the system is built based on the kinematic and dynamic equations. The kinematic and dynamic equations of the WMR are described as [12]:

$$\begin{cases} \dot{x}_M = \vartheta \cos\theta - \dot{\eta} \sin\theta \\ \dot{y}_M = \vartheta \sin\theta + \dot{\eta} \cos\theta \\ \dot{\theta} = \omega \end{cases} \tag{2}$$

$$M\dot{v} + Bv + Q\ddot{\gamma} + C\dot{\eta} + G\ddot{\eta} + \tau_d = \tau \tag{3}$$

where:

$$v = [\dot{\phi}_R \quad \dot{\phi}_L]^T, \quad \gamma = [\gamma_R \quad \gamma_L]^T, M = \begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}, m_{22} = m_{11}, m_{21} = m_{12},$$
$$m_{11} = m_G\left(\frac{r^2}{4} + \frac{ar^2}{4b^2}\right) + \frac{r^2}{4b^2}(I_G + 2I_D) + 2m_w r^2 + I_w, m_{12} = m_G\left(\frac{r^2}{4} - \frac{a^2r^2}{4b^2}\right) - \frac{r^2}{4b^2}(I_G + 2I_D)$$
$$B = m_G\frac{r^2}{2b}\omega\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2 & Q_1 \end{bmatrix}, Q_{1,2} = m_G\frac{r}{4}\left(1 \pm \frac{a^2}{b^2}\right) \pm \frac{r}{4b}(I_G + 2I_D), C = m_G\frac{r}{2}\omega\begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$G = m_G\frac{ar}{2b}\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

In which m_G is WMR body mass; m_w is each driving wheel mass; I_G is the inertial moment of the platform about the vertical axis through point G; I_D is the inertial moment of each wheel about its diameter axis and; I_w is the inertial moment of each wheel about its rotational axis. Formula (3) can be rewritten as (4) and (5):

$$\dot{v} = -M^{-1}Bv + M^{-1}\tau - M^{-1}(Q\ddot{\gamma} + C\dot{\eta}) + G\ddot{\eta} + \tau_d \tag{4}$$

$$\dot{v} = -M^{-1}Bv + M^{-1}\tau + d_2 \tag{5}$$

Where: $d_2 = -M^{-1}(Q\ddot{\gamma} + C\dot{\eta}) + G\ddot{\eta} + \tau_d$

For tracking control, define target point as $Z(x_Z, y_Z)$. The position error between point $M(x_M, y_M)$ and Z in OXY is as (6) [12]:

$$e_p = \begin{bmatrix} e_{p1} \\ e_{p2} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} x_Z - x_M \\ y_Z - y_M \end{bmatrix} \tag{6}$$

In (6) expresses the tracking error of the control system. The derivative of (6) along with time under the condition of wheels slip and external disturbances is obtained as (7):

$$\dot{e}_p = \begin{bmatrix} \dot{e}_{p1} \\ \dot{e}_{p2} \end{bmatrix} = hv + d_1 \tag{7}$$

where:

$$h = \begin{bmatrix} \left(\frac{e_{p2}}{b} - 1\right)\frac{r}{2} & -\left(\frac{e_{p2}}{b} + 1\right)\frac{r}{2} \\ -\frac{e_{p1}r}{2b} & \frac{e_{p1}r}{2b} \end{bmatrix}, d_1 = \begin{bmatrix} \left(\frac{\dot{\gamma}_R-\dot{\gamma}_L}{2b}\right)e_{p2} - \frac{\dot{\gamma}_R+\dot{\gamma}_L}{2} \\ -\left(\frac{\dot{\gamma}_R-\dot{\gamma}_L}{2b}\right)e_{p1} - \dot{\eta} \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} \dot{x}_Z \\ \dot{y}_Z \end{bmatrix}$$

Define variables $x_1 = e_p$; $x_2 = \dot{x}_1 + \lambda x_1$, where λ is a positive scalar. The time derivative of x_2 is:

$$\dot{x}_2 = \ddot{x}_1 + \lambda\dot{x}_1 \tag{8}$$

To be convenient to the system model build, (8) be rewritten to cancel the high order derivative of x_1. Substituting (5) and (7) into (8) leads to:

$$\begin{aligned} \dot{x}_2 &= h\dot{v} + \dot{h}v + \dot{d}_1 + \lambda(hv + d_1) \\ &= hM^{-1}Bv + hM^{-1}\tau + hd_2 + \dot{h}v + \dot{d}_1 + \lambda hv + cd_1 \\ &= P_1v + E\tau + d_3 \end{aligned} \tag{9}$$

in which:

$$P_1 = -hM^{-1}B, \quad E = hM^{-1}, \quad d_3 = hd_2 + \dot{h}v + \dot{d}_1 + \lambda hv + \lambda d_1$$

From (9) can be reformulated in the form of state variable. By substituting v from (7) into (9), $\dot{x}_2$ is written as (10):

$$\dot{x}_2 = Px_2 - \lambda Px_1 + E\tau + d \tag{10}$$

where:

$$P = P_1h^{-1} = M^{-1}B, \quad d = d_3 - h^{-1}d_1$$

From above calculation, the control model of the WMR will be established. The WMR state model can be expressed as (11):

$$\begin{cases} \dot{x}_1 = x_2 - \lambda x_1 \\ \dot{x}_2 = Px_2 - \lambda Px_1 + E\tau + d \end{cases} \tag{11}$$

In (11), λ is the positive constant with unknown exact value. For the good control performance, in this work, λ is viewed as unknown component which needs to be estimated. In this sense, define disturbances $D_1 = \lambda x_1$ and $D_2 = d - \lambda Px_1$. The state model (11) becomes:

$$\begin{cases} \dot{x}_1 = x_2 + D_1 \\ \dot{x}_2 = Px_2 + E\tau + D_2 \end{cases} \tag{12}$$

3. ADAPTIVE SLIDING MODE TRACKING CONTROLLER DESIGN FOR WMR

In this section, an adaptive SMC will be introduced for tracking control of the WMR. The control scheme is designed through two steps: the first one, the chattering free SMC is built based on the real disturbances. Next, the unknown disturbances will be estimated by a disturbance observer then the real disturbances will be replaced by estimated ones. The detail of each step is illustrated in the following subsections.

3.1. Disturbance observer based stable sliding surface design

Before designing the SMC, the sliding surface should be designed first. The sliding surface is defined as (13):

$$s = \dot{x}_2 + k_2x_2 + k_1x_1 \tag{13}$$

As the sliding surface is reached, $s = \dot{s} = 0$. Under this condition, replace (13) into (12) leads to the as (14):

$$\begin{cases} \dot{x}_1 = x_2 + D_1 \\ \dot{x}_2 = -k_2x_2 - k_1x_1 \end{cases} \tag{14}$$

In (14), if the mismatched disturbance D_1=0 then system states converge to the stable points. However, D1 is different from zero so from (14) the following result is obtained:

$$\ddot{x}_1 + k_2\dot{x}_1 + k_1x_1 = \dot{D}_1 + k_2D_1 \tag{15}$$

This means that the stability of x_1 can not be guaranteed because of the effect of the mismatched disturbance D_1. To overcome this drawback, the nonlinear disturbance observer [25] will be used to estimate the mismatched disturbances.

$$\begin{cases} \dot{\xi}_1 = \xi_2 - a_1(\xi_1 - x_1) + x_2 \\ \dot{\xi}_2 = -a_2\, tanh(b(\xi_1 - x_1)) \end{cases} \tag{16}$$

Where ξ_1 and ξ_2 are estimated values of x_1 and D_1, respectively; a_1, a_2, and b are positive scalars.

Define new variables $e_1 = \xi_1 - x_1$, $e_d = D_1 - \xi_2$; $e_2 = -e_d - a_1e_1$. Considering (14) and (16), the derivative of e_1 and e_2 is as (17):

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = -a_2\, tanh(be_1) - \dot{D}_1 - a_1e_1 \end{cases} \tag{17}$$

The convergence of e_1 and e_2 as well as e_d is given detail in [25]. Substituting (16) and (17) in to (15) and do some manipulations leading to as (18):

$$\ddot{x}_1 + k_2\dot{x}_1 + k_1x_1 = \dot{e}_d + k_2e_d \tag{18}$$

Because the estimated error e_d is bounded, it can be realized from (18) that if k_1 and k_2 are Hurwitz then the system states are stable. In considering the disturbances, the new sliding surface is established as (19):

$$s = \dot{x}_2 + k_2(x_2 + \xi_2) + k_1x_1 + \dot{\xi}_2 \tag{19}$$

3.2. Adaptive sliding mode control design

After ensuring that the sliding surface (19) is stable, in this section, the reaching controller will be designed for closed loop stable purpose. Introduce the control law [25]:

$$\tau = Q^{-1}(\tau_1 + \tau_2) \tag{20}$$

where:

$$\tau_1 = -Px_2 - k_2(x_2 + \xi_2) - k_1x_1 - \dot{\xi}_2 \tag{21}$$

and

$$\dot{\tau}_2 + \alpha\tau_2 = -(\varepsilon + k_3 + \mu)\, sgn(s) \tag{22}$$

in which $\alpha > 0$, $\varepsilon > |\dot{D}_2|$, $k_3 > \alpha|D_2|$, and μ is a positive scalar. Substituting (11) into (19):

$$s = Px_2 + E\tau + D_2 + k_2(x_2 + \xi_2) + k_1x_1 + \dot{\xi}_2 \tag{23}$$

The sliding surface (23) can be shortened by replacing the control law (20) with using (21) and (22) into (23). By this way, the sliding surface is obtained as (24):

$$s = \tau_2 + D_2 \tag{24}$$

The stability of the reaching controller is proven via Lyapunov theory. Choosing the Lyapunov function:

$$V = \frac{1}{2}s^Ts \tag{25}$$

Next, the time derivative of V will be proven to be negative. The time derivative of V along with (21) and (22) is as (26):

$$\begin{aligned}\dot{V} &= s^T\dot{s} = s^T(\dot{\tau}_2 + \dot{D}_2) = s^T(\dot{\tau}_2 + \alpha\tau_2 - \alpha\tau_2 + \dot{D}_2) \\ &= s^T[-(\varepsilon + k_3 + \mu)\, sgn(s) - \alpha\tau_2 + \dot{D}_2] \\ &= (-\varepsilon|s| + s^T\dot{D}_2) - (k_3s^T + \alpha s^T\tau_2) - \mu|s| \\ &< -\mu|s| \le 0\end{aligned} \tag{26}$$

This implies that the introduced controller (20) guarantees the reaching to the sliding surface of the system states as time goes to finite.

4. SIMULATION RESULT

In order to verify the effectiveness of the proposed control scheme, the simulations are setup in MATLAB/Simulink for a WMR with the following parameters:

$m_G = 10kg, m_w = 2kg, I_w = 0.1kgm^2, I_D = 0.05kgm^2, I_G = 4kgm^2,\ b = 0.3m,\ r = 0.15m,$ $a = 0.2m$

Parameters of the controller are given by: $\lambda = 10, k_1 = 10, k_2 = 10, \alpha = 100, \mu = 10, \varepsilon = 5.$
The simulations are executed in considering the input disturbances as (27):

$$\tau_d = [2 + sin(0.1t) \quad 1 + cos(0.1t)]^T \tag{27}$$

and the longitudinal and lateral slip factor can be chosen as (28):

$$[\dot{\gamma}_R, \dot{\gamma}_L, \dot{\eta}]^T = \begin{cases} [0,0,0]^T(m/s) & t < 2(s) \\ [0.3sin(2t), 0.3cos(2t), 0.2]^T(m/s) & t \geq 2(s) \end{cases} \quad (28)$$

The WMR is controlled to move with two kinds of reference trajectories: circular shape and trifolium shape. To demonstrate the effectiveness of the proposed adaptive sliding mode control (ASMC), this paper simulates the design algorithm along with the represented method in [6]. The simulation results for each case are illustrated in the following.

- Case 1: circle shape with the following equations of reference trajectory:

$$x_Z = 5\cos\left(\frac{\pi}{10}t\right), y_Z = 5\sin\left(\frac{\pi}{10}t\right) \quad (29)$$

The initial conditions of the mobile robot are given by: $[x_M(0), y_M(0), \theta(0)] = [5; -3.2; \pi/2]$.

The trajectory tracking of the ASMC and Department of Statistics Malaysia (DOSM) is depicted in Figure 2. It can be seen from Figure 2 that the responses of both ASMC and DOSM are good with similar trajectory at steady state time. However, at initial time, the response of the DOSM takes a long time to track the reference curve and has little oscillation at 3 s where the disturbances act on the system. This result is expressed thoroughly in Figure 3(a) which presents the tracking error in x-axis and y-axis. In Figure 3(a), the tracking errors of two controllers are almost the same in the y-axis with nearly zero steady state error. In the x-axis, the maximum tracking error of the proposed ASMC is about 0.05 m while this is bigger for DOSM (about 0.65 m). The advantages of the proposed ASMC and the DOSM are illustrated clearly in Figure 3(b), the control torques. In Figure 3(b), the control torques of the ASMC have a big pulse at initial, but they are smooth during the working time. Meanwhile, the control torques generated by the DOSM scheme are affected by chattering phenomena of the SMC, so they are highly oscillated in both transient time and steady state. This chattering is not good for power electronic devices as well as the mechanical systems. Also, at the time acting on the system, the control torques of the DOSM are too higher than those of the proposed ASMC.

- Case 2: the trifolium shape reference trajectory which is defined by (30):

$$x_Z = 5\cos\left(\frac{\pi}{10}t\right)\cos\left(\frac{\pi}{30}t\right), y_Z = 5\sin\left(\frac{\pi}{10}t\right)\sin\left(\frac{\pi}{30}t\right) \quad (30)$$

The initial conditions are: $[x_M(0), y_M(0), \theta(0)] = \left[5.1, \ -1.2, \ \frac{\pi}{2}\right]$.

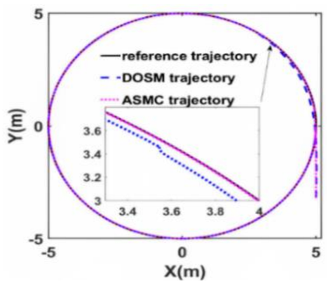
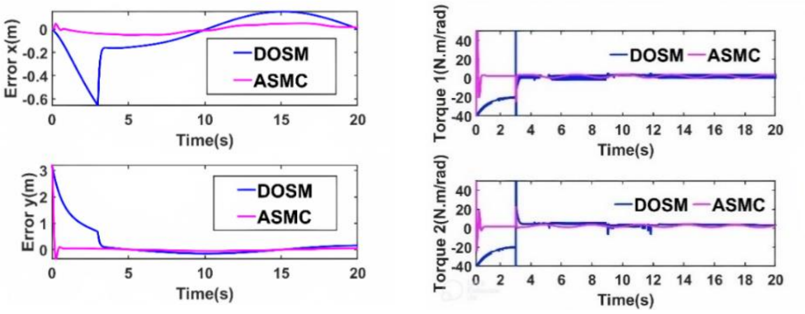


Figure 2. Trajectory tracking of the WMR in circular shape

The simulation results are presented in Figures 4 and 5. Same as case 1, the trajectory of the WMR in both controllers are exact and robust against unknown input disturbances and the unknown wheel slips. But the tracking errors of ASMC controller converge to zero quickly in Figure 4 (about 0.5 s with ASMC meanwhile DOSM about 9 s). In Figure 5(a), at the time where the orientation of the WMR changes a lot, the trajectory controlled by the proposed controller has lower tracking error than the DOSM controller. In the x-axis, the maximum error tracking of the proposed controller is 0.1 m and the DOSM controller is about 0.75 m (at the time the disturbances act on the system). Figure 5(b) is the comparison of the control torque between two control methods. It can be seen that the proposed control method is smoother, and the chattering problem is eliminated. Chattering reduction will increase the life of the actuator, this is an advantage of the proposed control method. Moreover, when the disturbances are acting on the system, the control torques of the DOSM exhibit an increase exceeding 40 Nm, whereas the proposed ASMC maintains these values below 20 Nm.



(a) (b)

Figure 3. Comparative response of tracking error and motor torqure, (a) trajectory tracking error in *X*, *Y* direction and (b) comparison of the wheel motor torque

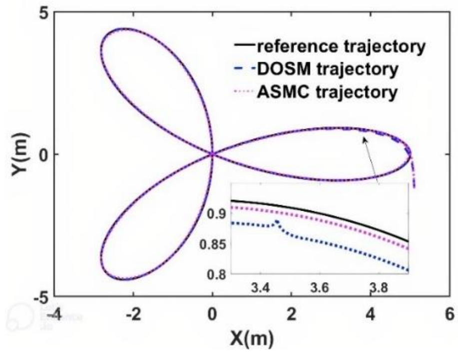
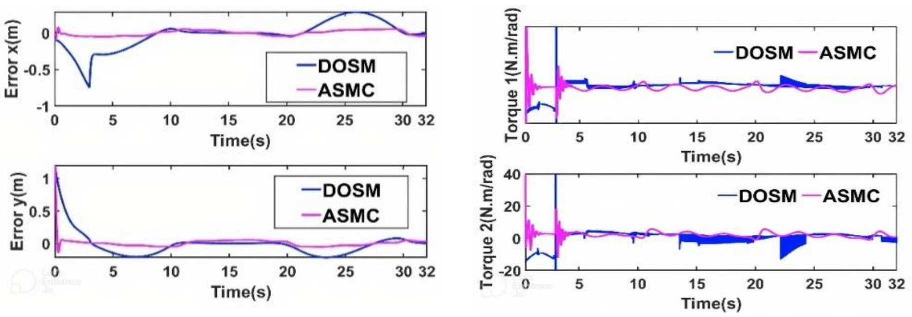


Figure 4. Trajectory tracking of the WMR in trifolium shape



(a) (b)

Figure 5. Comparative response of tracking error and motor torqure, (a) trajectory tracking error in *X*, *Y* direction and (b) comparison of the wheel motor torque

5. CONCLUSION

The paper introduces an ASMC algorithm for a WMR considering wheel slips, uncertainties, and input disturbances. The state space model of the system has been constructed, and an adaptive SMC scheme is developed using this model to ensure system stability. Then an observer is designed to estimate the unknown components in the system which enhances the overall performance of the scheme. With such a design approach, this methodology can be effectively extended to a class of non-linear uncertain models as well. In this paper, the stability of the overall system was mathematically proved based on the Lyapunov theory. Simulation examples have been presented two types of reference trajectory. In each simulation, the responses of the proposed controller were compared with another SMC. The simulation results showed that chattering the performances of the proposed ASMC are significantly improved.

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