Realization of fractional order lowpass filter using different approximation techniques

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ABSTRACT

Many research groups are starting to pay serious attention to the problem of fractional-order circuits. In this paper, a new approach to designing fractional order low-pass filter (FOLPF) is presented. Finding a rational approximation of the fractional Laplace operator s^α is a crucial step in the design of fractional order filters. A comparative study of the most widely used approximation techniques named continued fraction expansion (CFE) method and Biquadratic Approximation (RE) method is performed. Then the transfer function of the proposed FOLPF is calculated. Using operational amplifier, the proposed filter is synthesized. The proposed circuit is simulated using Texas instruments TINA software. The results obtained outperform the existing methods.

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1. INTRODUCTION

The simulation of the physical environment can make use of an extension of classic integral and differential equations known as fractional order differential and integral equations [1]. The fractional differential equations could not be solved using any of the methods that are available, so the integer-order models were used instead. Several different approaches are now being utilised in the process of developing applications predicated on fractional derivatives and integration. It is possible to improve the characterization of a wide variety of dynamic systems by employing non-integer order dynamic models that are founded on fractional calculus, differentiation, or integration. Calculus in its classical form is built on the foundation of differentiation and integration of integer orders [1]. Fractional order systems makes use of fractional order differentiation and integration as constituent elements. Recent applications of fractional order systems in electrical engineering is an appreciable thing. A fractional order element (also called as fractance device) is designated with s^α.

The design of fractional order filters requires a number of steps, one of which is the realization of the fractance device. Krishna and Reddy [2] discussed a realization of a fractance device that is based on a continuing fraction growth. The realization of a fractional impedance of order 1/2 is made possible by Hilbert's structure, which is also put to use in the construction of fractional order controllers [3]. Many of the electrical systems make use of something called a fractional order filter. The literature contains many examples of very well built low pass, high pass, band pass, band reject, and all pass fractional order filters, as well as first and second order pass filters. They are built to operate in both the current mode and the voltage mode simultaneously. In the design of a fractional-order filter, some of the many active elements that are commercially available and can be used include operational amplifiers, differential difference current conveyors (DDCCs), current feedback operational amplifiers (CFOAs), second generation current conveyors (CCIIs), operational transconductance amplifiers (OTAs), and current amplifiers (CAs) [4]. To generate a transfer function that is analogous to that of a fractional-order filter, one may employ any one of a number of different topologies. The topologies follow the leader feedback (FLF), inverse follow the leader feedback (IFLF), or single amp biquad (SAB) in conjunction with a first-order filter are the ones that are utilised the most frequently. Even if there are a variety of topologies available, the end user must make use of a specific topology in order to satisfy the design requirements.

Dorčák *et al.* [5] discussed the design and electronic implementation of the fractional-order controller and controlled system. This system is based on the Laplace transfer function of the corresponding electronic circuit being equivalent to the Laplace transfer function of the original fractional order systems. Mahmood *et al.* [6] purposed to synthesise a fractance circuit in an electronic circuit to achieve fractional order derivatives. Two different types of fractances were synthesised and linked in an operational amplifier as passive RC circuits to achieve the realization. Iqbal and Shekh [7] have studied about different types of approximations of fractional order systems. Realization of fractional order filters at microwave region is studied in [8]. Tewari and Arya [9] have designed and performed the PSPICE based simulation of operational transresistance amplifier (OTRA) based fractional low pass filter of order 1.5. Resistor less realization of generalized filters of fractional order using OTAs as active elements is realized in [10]. Operational amplifier based design of fractional order controllers is studied using simulation tools [11]. Sacu and Alci [12] used Cadence-PSpice to build and simulate simple OTA-based fractional order low-pass and high-pass filters of order $(n + \alpha)$, where $0 < \alpha < 1$ and $n \geq 1$. For any order $0 < \alpha < 1$, the fractional-order low-pass filter based on a single fractance element is represented in [13] by using optimal integer-order transfer function approximations. Initially, integer-order filter coefficients for the FLF are found directly in the range of 0.01 to 0.99 with a step size of 0.01. This is done by employing the colliding bodies optimisation technique. A novel rational approximation for the fractional order operator is proposed in [14]. Signal flow graphs synthesise OTA based fractional element in [15]. Theory, simulation, and approximation function match. Genetic algorithm is used in the synthesis and optimization of fractional order elements [16]. Different passive realization of fractional order capacitors studied in [17].

In order to design the fractional order low-pass filter (FOLPF), it is necessary to approximate the operator to a certain degree on a numerical scale. These methodologies for approximation were proposed by the researchers at various points in time throughout the study. Based on a general filter topology, the work that was done in [18] introduces fractional order low-pass, high-pass, band-pass, and band-reject filter responses at the same circuit utilising low-voltage active elements. Simulation of the designed circuits is performed with Spice software and makes use of the properties of 0.35 µm Taiwan semiconductor manufacturing company (TSMC) complementary metal-oxide semiconductor (CMOS) technology. This article presents the results of a comparative examination of the many rational approximation approaches that are currently available. Yüce and Tan [19] proposed a basic way for generating fractional-order filters with transfer functions by making use of Laplace operators of a variety of fractional orders. Mishra *et al.* [20] have come up with a proposal for a fractional order butterworth lowpass filter that makes use of differential voltage current conveyor.

A discussion of a proven example of the realization of a fractional order element through the use of the partial fraction expansion method in [21], [22]. Theirs work compares two different approaches to design by imitating skin tissue and using AgCl electrodes in order to test and evaluate electrical bio-impedance circuits and systems [23]. The models are built on fractional-order elements, implemented with active components, and may reflect bio-impedance characteristics up to 10 kHz. A fractional-order passive RC low-pass filter is investigated in [24]. The progression of behavior in the temporal realm was revealed through fractional orders. MATLAB's output consisted of low-order fractional-order filters. The innovative method of curve fitting provides a close approximation of this function. Using a universal fractional-order transfer function, this method provides an explanation for all of the many types of fractional-order filters. A fractional-order element analogue modelling system is illustrated in [25], which may be found here. The traditional circuit theory is applied here. Mijat *et al.* [26] discusses a number of different processes that can be used to make the fractional element. A fractional change occurs in the simulated frequency response. A single OTRA is utilised in the construction of an active current mode fractional order analogue filter. The 1.5-order filter that has been proposed. The findings of the PSpice simulation matched the theoretical predictions. The RC-RC decomposition was utilised in the filter that was manufactured by OTRA. Prommee *et al.* [27] demonstrate the application of fractional order devices in the processing of bio-medical signals employing OTAs. These amplifiers work in conjunction with the fractional order devices. Research by Krishna [28] provides an indepth analysis of the many developments that have been made in the realm of realising the fractance device. A variety of different comparable circuit representations of fractional order elements are given by [29], [30]. Narayan *et al.* [31] present a realization of a fractional order bandpass filter by employing a reconfigurable device. The paper is organised as follows: different rational approximation methods are discussed in section 2, section 3 deals with the FOLPF, section 4 discusses the implementation of the filter, the finalized circuit and the corresponding results are presented in section 5, and finally conclusions are drawn in section 6.

2. RATIONAL APPROXIMATION METHODS

Given that the fractional order system is in fact an infinite order system, it is preferable to approximate lengths with finite values. Both the Laplace and the z domains can be used to make an approximation of the fractional order operator known as s^{α}. Approximations that fall within the s-domain umbrella include the oustaloup approach, the mastudas method, and the least square method. Each approach has a number of advantages and disadvantages. The continued fraction expansion (CFE) method and Reyad Elkhazali (RE) approximation are both taken into consideration in this article as potential methods of approximation. Because of the complexity of the hardware, only approximation up to the second order is taken into consideration. It can be written as (1) [4], which is the second order approximation of s^{α}.

$$s^{\alpha} = \frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} \tag{1}$$

where a_0, a_1, a_2 are the filter coefficients.

2.1. Continued fraction expansion method

The CFE of $(1+x)^{\alpha}$ is used to characterise this technique [4], [32].

$$(1+x)^{\alpha} = \frac{1}{1-} \frac{\alpha x}{1+} \frac{(1+\alpha)x}{2+} \frac{(1-\alpha)x}{3+} \frac{(2+\alpha)x}{2+} \frac{(2-\alpha)x}{5+.....} \tag{2}$$

when $x = s - 1$ is substituted, the above CFE converges down the negative real axis from $s = -\infty$ to $s = 0$ in the finite complex s-plane. Considering the first four terms of the expansion a second order approximation will be derived with the coefficients as (3):

$$\begin{aligned} a_0 &= \alpha^2 + 3\alpha + 2 \\ a_1 &= 8 - 2\alpha^2 \\ a_2 &= \alpha^2 - 3\alpha + 2 \end{aligned} \tag{3}$$

2.2. Biquadratic Approximation method

Khazali *et al.* [14] presented this approximation in 2019. This method is designated as RE to designate the name of the scientist. A biquadratic approximation approach can be used to estimate a fractance operator. The rational approximation will take the shape of a cascade connection of biquadratic transfer functions as (4):

$$\left(\frac{s}{\omega_g}\right)^{\alpha} = \prod_{i=1}^{n} H_i\left(\frac{s}{\omega_i}\right) = \prod_{i=1}^{n} \frac{N_i\left(\frac{s}{\omega_i/\omega_g}\right)}{D_i\left(\frac{s}{\omega_i/\omega_g}\right)} \tag{4}$$

where

$$\begin{aligned} \omega_i &= Center\,Frequency \\ \omega_g &= Geometric\,Mean \end{aligned} \tag{5}$$

The remaining center frequencies may be computed using a recursive formula using ω_1 as in (6):

$$\omega_i = \omega_x^{2(i-1)} \omega_1, \;\; i = 2, 3, 4, ..., n. \tag{6}$$

ω_x is calculated by solving in (7):

$$a_0 a_2 \eta \gamma^4 + a_1(a_2 - a_0)\gamma^3 + (a_1^2 - a_2^2 - a_0^2)\eta\gamma^2 + a_1(a_2 - a_0)\gamma + a_0 a_2 \eta = 0 \tag{7}$$

where $\eta = tan(\frac{\alpha\pi}{4})$. The expression for the quadratic will be (8):

$$\left(\frac{s}{\omega_g}\right)^{\alpha} \cong \frac{a_0\left(\frac{s}{\omega_i}\right)^2+a_1\left(\frac{s}{\omega_i}\right)+a_2}{a_2\left(\frac{s}{\omega_i}\right)^2+a_1\left(\frac{s}{\omega_i}\right)+a_0}, \qquad i=1,2,3,... \tag{8}$$

The values of the coefficients a_0, a_1, a_2 are calculated in (9):

$$\begin{aligned} a_0 &= \alpha^{\alpha}+2\alpha+1 \\ a_2 &= \alpha^{\alpha}-2\alpha+1 \\ a_1 &= (a_2-a_0)\tan\left(\frac{(2+\alpha)\pi}{4}\right) \end{aligned} \tag{9}$$

After calculating the coefficients, the rational approximation for s^{α} may be determined. The second order rational approximations obtained for different values of α is as shown in Table 1.

Table 1. Rational approximation of second order

α	$H_{RE}(s)$	$H_{CFE}(s)$
0.1	$\frac{1.994s^2+5.0828s+1.594}{1.594s^2+5.082s+1.994}$	$\frac{2.31s^2+7.98s+1.71}{1.71s^2+7.98s+2.31}$
0.2	$\frac{2.125s^2+5.051s+1.325}{1.325s^2+5.051s+2.125}$	$\frac{2.64s^2+7.92s+1.44}{1.44s^2+7.92s+2.64}$
0.3	$\frac{2.297s^2+4.998s+1.097}{1.097s^2+4.998s+2.297}$	$\frac{2.99s^2+7.82s+1.19}{1.19s^2+7.82s+2.99}$
0.4	$\frac{2.493s^2+4.924s+0.8931}{0.8931s^2+4.924s+2.493}$	$\frac{3.36s^2+7.68s+0.96}{0.96s^2+7.68s+3.36}$
0.5	$\frac{2.707s^2+4.828s+0.7071}{0.7071s^2+4.828s+2.707}$	$\frac{3.75s^2+7.5s+0.75}{0.75s^2+7.8s+3.75}$
0.6	$\frac{2.936s^2+4.71s+0.536}{0.536s^2+4.71s+2.936}$	$\frac{4.16s^2+7.28s+0.56}{0.56s^2+7.28s+4.16}$
0.7	$\frac{3.179s^2+4.569s+0.3791}{0.3791s^2+4.569s+3.179}$	$\frac{4.59s^2+7.02s+0.39}{0.39s^2+7.02s+4.59}$
0.8	$\frac{3.437s^2+4.404s+0.2365}{0.2365s^2+4.404s+3.347}$	$\frac{5.04s^2+6.72s+0.24}{0.24s^2+6.72s+5.04}$
0.9	$\frac{3.71s^2+4.215s+0.1095}{0.1095s^2+4.215s+3.71}$	$\frac{5.51s^2+6.38s+0.11}{0.11s^2+6.38s+5.51}$

A comparison of the two rational approximations (for $\alpha = 0.4$) is as shown in Figures 1 and 2. The phase response offered by the RE approximation is good as compared to CFE based method. From the magnitude response it is observed that the CFE based method produces good results for larger range of frequencies as compared to RE method. Keeping in view of the better linearity in phase angle, RE method is considered for the implementation.

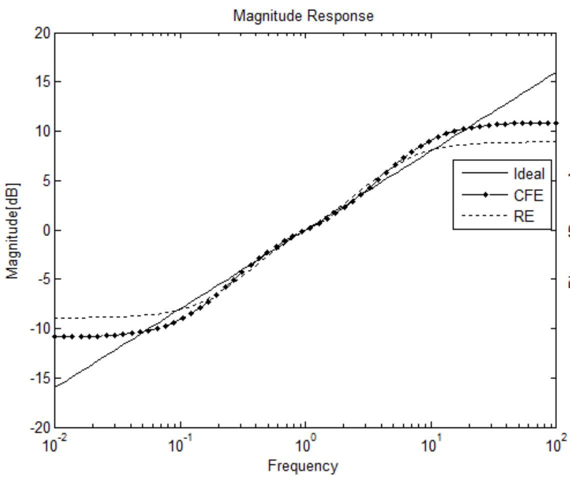

Figure 1. Comparison for magnitude response of different approximation methods

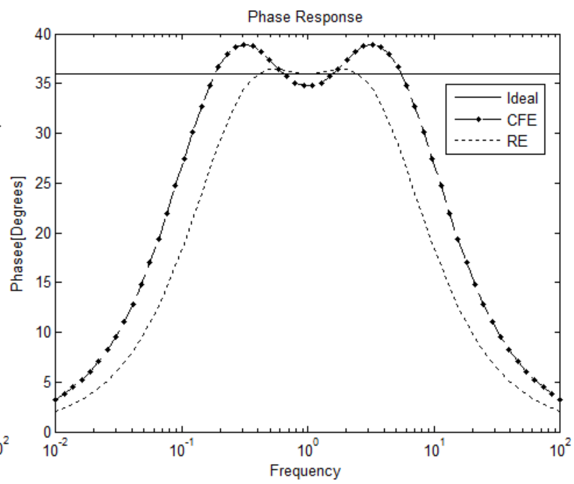

Figure 2. Comparison for phase response of different approximation methods

3. FRACTIONAL ORDER LOW-PASS FILTER

Fractional order filters can meet the exact design criteria, but traditional integer-order filters were unable to do so. Comparing the design parameters to the integer-order filters, the fractional notion increases the level of safety a little amount. Fractional order filters make it simple to meet the requirements for the circuit design and tuning parameters. Only fractional order filters make it feasible to vary and modify parameters like the roll-off frequency to any desired slope. Thus, compared to integer-order filters, fractional order filters offer greater design freedom.

A standard filter and a fractional-order filter vary in that additional parameters are included in the term that determines the slope of attenuation of a given transfer function. Therefore, the equation for a fractional-order filter's attenuation of transfer function is $20(n+\alpha)$ (dB/decade). Where n is an unsigned integer number, often between 1 and 10, and α is defined as a real number in the range [0 1]. The transfer function of FOLPF is given by (10) [24]:

$$H_{n+\alpha}^{LP} = \frac{k_1}{s^\alpha (s^n + k_2) + k_3} \tag{10}$$

where n is an integer and $0 < \alpha < 1$. The following are the values of k's as (11):

$$\begin{aligned} k_1 &= 1 \\ k_2 &= 1.1796\alpha^2 + 0.167\alpha + 0.21735 \\ k_3 &= 0.19295\alpha + 0.81369 \end{aligned} \tag{11}$$

Substituting the second order approximation of the fractional order operator in the transfer function of the low-pass filter, the equation becomes as (12):

$$H_{n+\alpha}^{LP} = \frac{k_1}{\left(\frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0}\right)(s^n + k_2) + k_3} \tag{12}$$

After rearranging the terms, the equation becomes as (13):

$$H_{n+\alpha}^{LP} = \frac{k_1\left(a_2 s^2 + a_1 s + a_0\right)}{\left(\begin{array}{l} a_0 s^{n+2} + a_1 s^{n+1} + a_2 s^n + s^2\left(k_3 a_2 + k_2 a_0\right) \\ +s\left(k_3 a_1 + k_2 a_1\right) + \left(k_3 a_0 + k_2 a2\right) \end{array}\right)} \tag{13}$$

For $n = 1$, in (13) simplifies to (14):

$$H_{1+\alpha}^{LP} = \frac{k_1\left(a_2 s^2 + a_1 s + a_0\right)}{a_0\left(s^3 + c_1 s^2 + c_2 s + c_3\right)} \tag{14}$$

where,

$$\begin{aligned} c_1 &= \left(k_3 a_2 + k_2 a_0 + a_1\right)/a_0 \\ c_2 &= \left(k_3 a_1 + k_2 a_1 + a_2\right)/a_0 \\ c_3 &= (k_3 a_0 + k_2 a2)/a_0 \end{aligned} \tag{15}$$

The equations indicate that the values of the filter coefficients are dependent upon the value of fractional order α. Table 2 depict the values of filter coefficients for different value of fractional order. The transfer functions of the FOLPF using different rational approximation techniques (for different values of α) as tabulated in Table 3.

Table 2. Values of filter co-efficients for different values of α

α	a_0	a_1	a_2	c_1	c_2	c_3
0.1	2.31	7.98	1.71	3.497325	0.9053559	0.04038944
0.2	2.64	7.92	1.44	3.101633	0.9029765	0.08254491
0.3	2.99	7.82	1.19	2.794686	0.9580753	0.1200770
0.4	3.36	7.68	0.96	2.563302	1.046208	0.1501903
0.5	3.75	7.5	0.75	2.397695	1.149750	0.1721550
0.6	4.16	7.28	0.56	2.29044	1.255711	0.1864237
0.7	4.59	7.02	0.39	2.235792	1.354332	0.1941091
0.8	5.04	6.72	0.24	2.229228	1.438158	0.1966716
0.9	5.51	6.38	0.11	2.267138	1.501410	0.1957304

Table 3. Transfer functions of FOLPF

α	$H_{LPCFE}(s)$	$H_{LPRE}(s)$
0.1	$\frac{1.71s^2+7.98s+2.31}{2.31s^3+8.0788s^2+2.0914s+0.0933}$	$\frac{1.594s^2+5.0820s+1.9940}{1.9940s^3+5.1696s^2+1.8369s+0.0839}$
0.2	$\frac{1.44s^2+7.92s+2.64}{2.64s^3+8.1883s^2+2.3839s+0.2179}$	$\frac{1.325s^2+5.051s+2.1250}{2.1250s^3+5.2734s^2+1.9269s+0.1888}$
0.3	$\frac{1.19s^2+7.82s+2.99}{2.99s^3+8.3561s^2+2.8646s+0.359}$	$\frac{1.097s^2+4.998s+2.297}{2.297s^3+5.4204s^2+2.1973s+0.3044}$
0.4	$\frac{0.96s^2+7.68s+3.36}{3.36s^3+8.6127s^2+3.5153s+0.5046}$	$\frac{8.931s^2+4.9240s+2.493}{2.493s^3+5.63s^2+2.5314s+0.4206}$
0.5	$\frac{0.75s^2+7.5s+3.75}{3.75s^3+8.9914s^2+4.3116s+0.6456}$	$\frac{7.071s^2+4.8280s+2.707}{2.707s^3+5.9205s^2+2.9998s+0.5287}$
0.6	$\frac{0.56s^2+7.28s+4.16}{4.16s^3+9.5282s^2+5.2238s+0.7755}$	$\frac{0.536s^2+4.71s+2.9360}{2.9360s^3+6.3130s^2+3.5533s+0.6212}$
0.7	$\frac{0.39s^2+7.02s+4.59}{4.59s^3+10.2623s^2+6.2164s+0.8910}$	$\frac{3.791s^2+4.569s+3.179}{3.179s^3+6.8293s^2+4.1712s+0.6928}$
0.8	$\frac{0.24s^2+6.72s+5.04}{5.04s^3+11.2353s^2+7.2483s+0.9912}$	$\frac{0.2365s^2+4.4040s+3.437}{3.437s^3+7.4944s^2+4.8294s+0.7407}$
0.9	$\frac{0.11s^2+6.38s+5.51}{5.51s^3+12.4919s^2+8.2728s+1.0785}$	$\frac{0.1095s^2+4.215s+3.71}{3.71s^3+8.3364s^2+5.5023s+0.7653}$

4. IMPLEMENTATION OF THE PROPOSED FILTER

In order to implement the transfer functions derived in the previous sections, FLF topology is used. In this paper, feed-forward path based FLF structure is used for implementation. The outputs are available in terms of $s^n V_{out}, s^{n-1} V_{out}, ..., sV_{out}, V_{out}$. These outputs are used in a feed-forward form to get n^{th} order filter with arbitrarily transmission zeros as shown in Figure 3. In this structure, the outputs from the lossless integrators are multiplied by coefficients and then added together to get final output [33], [34].

$$\frac{V_1}{V_{in}} = H_1(s) = \frac{1}{s^n + k_1 s^{n-1} + k_2 s^{n-2} .....k_{n-1}s + k_n} \tag{16}$$

$$\frac{V_{out}}{V_1} = H_2(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} .....a_{n-1}s + a_n \tag{17}$$

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= H(s) = H_1(s)H_2(s) \\ &= \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} .....a_{n-1}s + a_n}{s^n + k_1 s^{n-1} + k_2 s^{n-2} .....k_{n-1}s + k_n} \end{aligned} \tag{18}$$

The above transfer function can be realized using only lossless integrators and multipliers.

$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_1}\frac{V_1}{V_{in}} = H_1 H_2 = \frac{As^2 + Bs + C}{s^3 + Ds^2 + Es + F} \tag{19}$$

Now splitting the transfer function.

$$\begin{aligned} & H_1 = \frac{V_{out}}{V_1} = As^2 + Bs + C \\ & \Rightarrow V_{out} + (-As^2 V_1 - BsV_1 - CV_1) = 0 \\ & \Rightarrow V_{out} + (-s^2 V_1 \tfrac{1}{1/A} - sV_1 \tfrac{1}{1/B} - V_1 \tfrac{1}{1/C}) = 0 \end{aligned} \tag{20}$$

$$H_2 = \frac{V_1}{V_{in}} = \frac{1}{s^3 + Ds^2 + Es + F} \tag{21}$$

The above equation can be rearranged as (22):

$$\begin{aligned} & V_{in} - (s^3 + Es)V_1 - (Ds^2 + F)V_1 = 0 \\ & \Rightarrow V_{in} - \left(s^3 + s\frac{1}{1/E}\right) V_1 - \left(s^2 \frac{1}{1/D} + \frac{1}{1/F}\right) V_1 = 0 \end{aligned} \tag{22}$$

The operational amplifier based realization valid for any fractional order α is as shown in Figure 4.

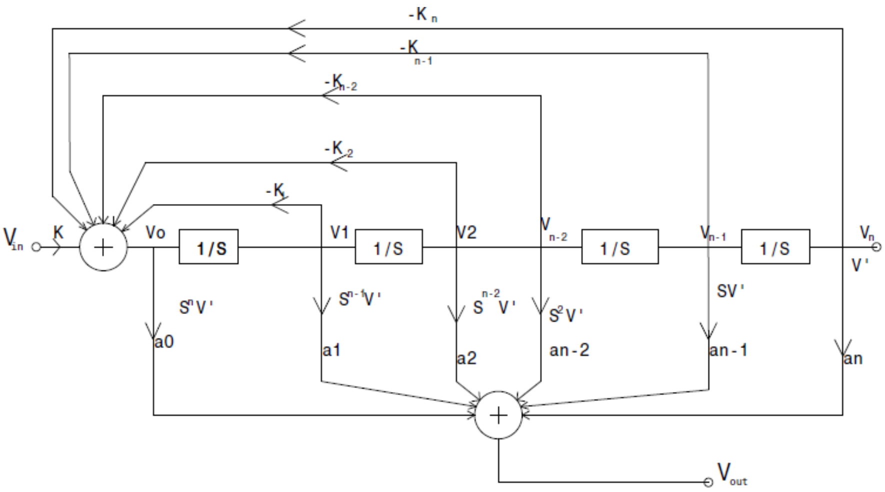

Figure 3. Feed forward path based FLF

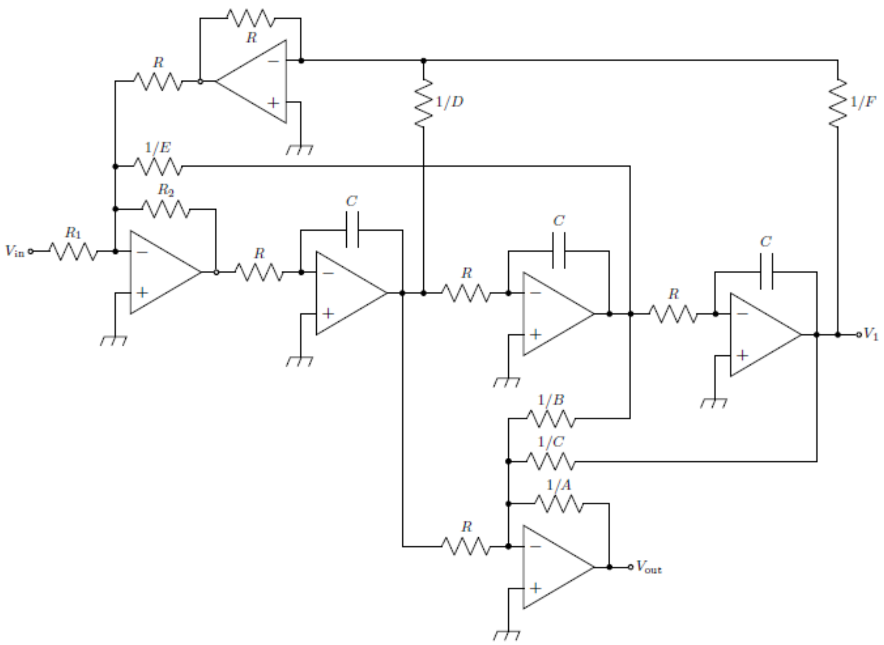

Figure 4. Synthesized circuit for any value of α

5. RESULTS

To illustrate the methodology, a value of $\alpha = 0.7$ is considered. In (23) is the transfer function of RE approximation is taken into consideration.

$$H^{LP}_{1.7}(s) = \frac{3.791s^2+4.569s+3.179}{3.179s^3+6.8293s^2+4.1712s+0.6928} \tag{23}$$

The circuit realized is as shown in Figure 5. If frequency denormalization by a factor of 20k rad/sec and impedance scaling by a value of 10000, then all resistances and capacitance values will be practicable. The same circuit is used for the simulation purpose using TINA software. For the simulation purpose Texas instruments TINA software is used. The corresponding results of transient response and bode plot are as shown in Figures 6 and 7 respectively.

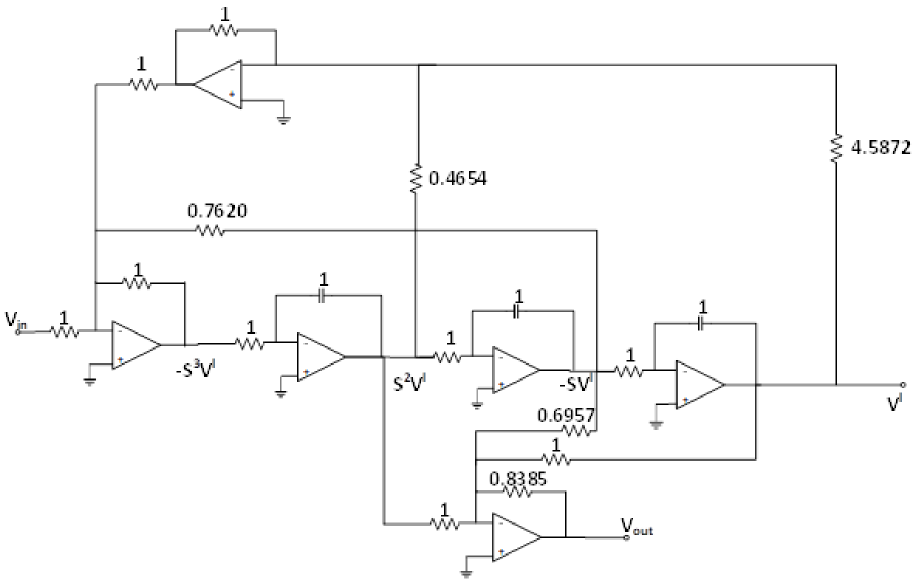

Figure 5. Realized circuit for $\alpha = 0.7$ using RE approximation

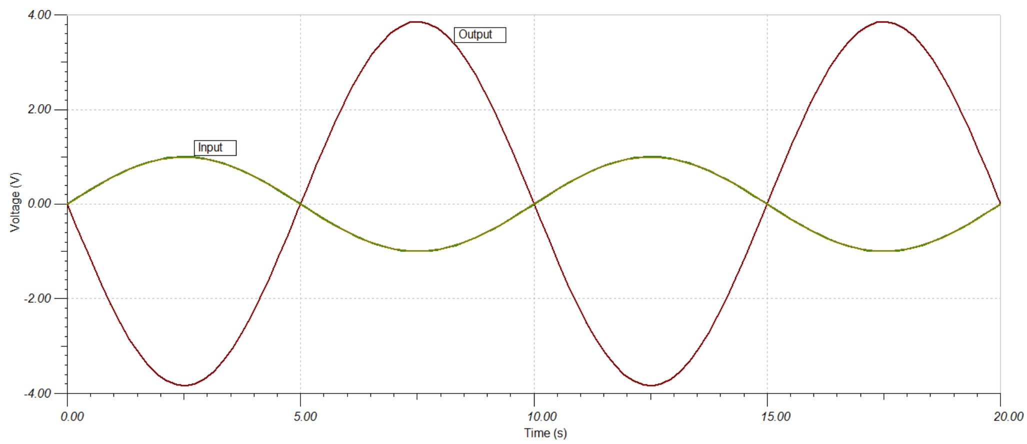

Figure 6. Transient response

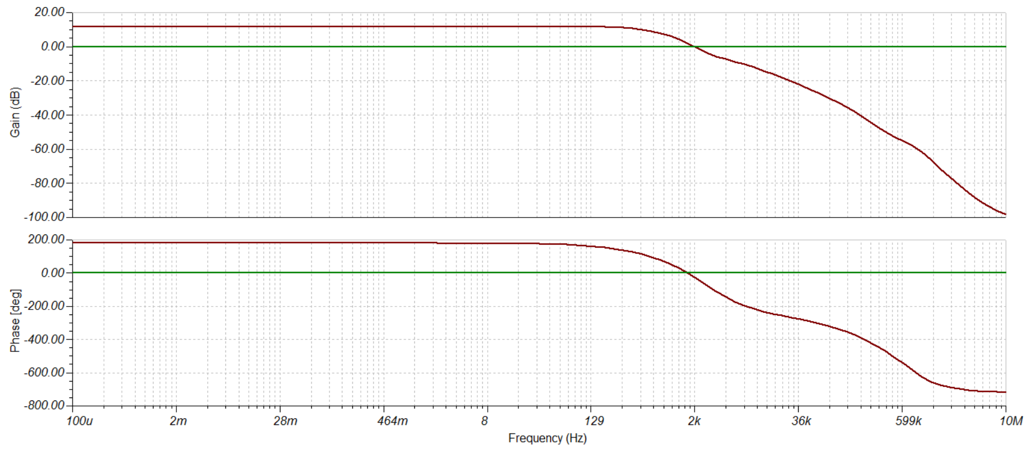

Figure 7. Bode plot

6. CONCLUSION

This paper presents a novel realization of FOLPF using different approximation techniques. Initially, a comparative study of the existing approximation techniques for fractional order device is carried out. Then synthesis of the FOLPF is carried out using FLF topology. The proposed method is straight forward and makes use of an operational amplifier for the realization. This method makes use of only lossless integrators and amplifiers. A 100 mHz and 1 V_{PP} sinusoidal signal is chosen as input. LM301A operational amplifier is chosen for simulation purpose. It is observed that there is a phase shift of 180 between input and output. The linear range of operation can be up to 100 μHz to 2 KHz. So, the procedure suggested can be used for the realization of fractional order filters.

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REFERENCES

[1] K. B. Oldham and J. Spanier, *The Fractional Calculus*. New York, London: Academic Press, 1974.

[2] B. T. Krishna and K. V. V. S. Reddy, “Active and passive realization of fractance device of order ½,” *Active and Passive Electronic Components*, vol. 2008, pp. 1–5, 2008, doi: 10.1155/2008/369421.

[3] T. C. Haba, G. L. Loum, J. T. Zoueu, and G. Ablart, “Use of a component with fractional Impedance in the realization of an analogical regulator of order 1/2,” *Journal of Applied Sciences*, vol. 8, no. 1, pp. 59–67, 2007, doi: 10.3923/jas.2008.59.67.

[4] B. T. Krishna, “Studies on fractional order differentiators and integrators: a survey,” *Signal Processing*, vol. 91, no. 3, pp. 386–426, 2011, doi: 10.1016/j.sigpro.2010.06.022.

[5] Ľ. Dorčák, J. Valsa, E. Gonzalez, J. Terpák, I. Petráš, and L. Pivka, “Analogue Realization of Fractional-Order Dynamical Systems,” *Entropy*, vol. 15, no. 12, pp. 4199–4214, 2013, doi: 10.3390/e15104199.

[6] A. K. Mahmood, S. Abdul, and R. Saleh, “Realization of Fractional Order Differentiator By Analogue Electronic Circuit,” *International Journal of Advances in Engineering & Technology*, vol. 8, pp. 1939–1951, 2015.

[7] A. Iqbal and R. R. Shekh, “A comprehensive study on different approximation methods of Fractional order system,” *International Research Journal of Engineering and Technology (IRJET)*, vol. 3, no. 8, pp. 1848–1853, 2016.

[8] D. K. Upadhyay and S. Mishra, “Fractional order microwave lowpass-bandpass filter,” in *2015 Annual IEEE India Conference (INDICON)*, 2015, pp. 1–5, doi: 10.1109/INDICON.2015.7443282.

[9] P. K. Tewari and R. Arya, “Design and Realization of Fractional Low-Pass Filter of 1.5 Order Using a Single Operational Transresistance Amplifier,” *International Journal of Signal Processing, Image Processing and Pattern Recognition*, vol. 9, no. 9, pp. 69–76, 2016, doi: 10.14257/ijsip.2016.9.9.07.

[10] G. Tsirimokou, C. Psychalinos, and A. S. Elwakil, “Fractional-order electronically controlled generalized filters,” *International Journal of Circuit Theory and Applications*, vol. 45, no. 5, pp. 595–612, 2017, doi: 10.1002/cta.2250.

[11] A. K. Mahmood, S. Abdul, and R. Saleh, “Realization of fractional-order proportional-integral-derivative controller using fractance circuit,” *Jea Journal of Electrical Engineering*, vol. 2, no. 1, pp. 22–32, 2018.

[12] I. E. Sacu and M. Alci, “Low-power OTA-C based tuneable fractional order filters,” *Informacije MIDEM*, vol. 48, no. 3, pp. 135–144, 2018.

[13] S. Mahata, S. K. Saha, R. Kar, and D. Mandal, “Approximation of fractional-order low-pass filter,” *IET Signal Processing*, vol. 13, no. 1, pp. 112–124, 2019, doi: 10.1049/iet-spr.2018.5128.

[14] R. E. -Khazali, I. M. Batiha, and S. Momani, “Approximation of Fractional-Order Operators,” in *Fractional Calculus*, vol. 303, Singapore: Springer, 2019, pp. 121–151, doi: 10.1007/978-981-15-0430-3_8.

[15] P. Prommee, N. Wongprommoon, and R. Sotner, “Frequency Tunability of Fractance Device based on OTA-C,” in *2019 42nd International Conference on Telecommunications and Signal Processing (TSP)*, 2019, pp. 76–79, doi: 10.1109/TSP.2019.8768816.

[16] A. Kartci *et al.*, “Synthesis and Optimization of Fractional-Order Elements Using a Genetic Algorithm,” *IEEE Access*, vol. 7, pp. 80233–80246, 2019, doi: 10.1109/ACCESS.2019.2923166.

[17] M. S. Semary, M. E. Fouda, H. N. Hassan, and A. G. Radwan, “Realization of fractional-order capacitor based on passive symmetric network,” *Journal of Advanced Research*, vol. 18, pp. 147–159, 2019, doi: 10.1016/j.jare.2019.02.004.

[18] I. E. Saçu and M. Alçi, “A current mode design of fractional order universal filter,” *Advances in Electrical and Computer Engineering*, vol. 19, no. 1, pp. 71–78, 2019, doi: 10.4316/AECE.2019.01010.

[19] A. Yüce and N. Tan, “Electronic realisation technique for fractional order integrators,” *The Journal of Engineering*, vol. 2020, no. 5, pp. 157–167, 2020, doi: 10.1049/joe.2019.1024.

[20] S. K. Mishra, M. Gupta, and D. K. Upadhyay, “Active realization of fractional order Butterworth lowpass filter using DVCC,” *Journal of King Saud University - Engineering Sciences*, vol. 32, no. 2, pp. 158–165, 2020, doi: 10.1016/j.jksues.2018.11.005.

[21] V. Alimisis, C. Dimas, G. Pappas, and P. P. Sotiriadis, “Analog Realization of Fractional-Order Skin-Electrode Model for Tetrapolar Bio-Impedance Measurements,” *Technologies*, vol. 8, no. 4, pp. 1–28, 2020, doi: 10.3390/technologies8040061.

[22] B. T. Krishna, “Realization of Fractance Device using Fifth Order Approximation,” *Communications on Applied Electronics*, vol. 7, no. 34, pp. 1–5, 2020, doi: 10.5120/cae2020652869.

[23] V. Alimisis, C. Dimas, G. Pappas, and P. P. Sotiriadis, “Analog Realization of Fractional-Order Skin-Electrode Model for Tetrapolar Bio-Impedance Measurements,” *Technologies*, vol. 8, no. 4, p. 61, 2020, doi: 10.3390/technologies8040061.

[24] K. Biswal, M. C. Tripathy, and S. Kar, “Performance Analysis of Fractional Order Low-pass Filter,” in *Advances in Intelligent Computing and Communication*, vol. 109, Singapore: Springer, 2020, pp. 224–231, doi: 10.1007/978-981-15-2774-6_28.

[25] S. Kapoulea, C. Psychalinos, and A. S. Elwakil, "FPAA-Based Realization of Filters with Fractional Laplace Operators of Different Orders," *Fractal and Fractional*, vol. 5, no. 4, pp. 1–10, 2021, doi: 10.3390/fractalfract5040218.

[26] N. Mijat, D. Jurisic, and G. S. Moschytz, "Analog Modeling of Fractional-Order Elements: A Classical Circuit Theory Approach," *IEEE Access*, vol. 9, pp. 110309–110331, 2021, doi: 10.1109/ACCESS.2021.3101160.

[27] P. Prommee, P. Pienpichayapong, N. Manositthichai, and N. Wongprommoon, "OTA-based tunable fractional-order devices for biomedical engineering," *AEU - International Journal of Electronics and Communications*, vol. 128, pp. 1–40, 2021, doi: 10.1016/j.aeue.2020.153520.

[28] B. T. Krishna, "Recent Developments on the Realization of Fractance Device," *Fractional Calculus and Applied Analysis*, vol. 24, no. 6, pp. 1831–1852, 2021, doi: 10.1515/fca-2021-0079.

[29] S. Holm, T. Holm, and Ø. G. Martinsen, "Simple circuit equivalents for the constant phase element," *PLOS ONE*, vol. 16, no. 3, pp. 1–12, 2021, doi: 10.1371/journal.pone.0248786.

[30] Y. Wei, Y. Chen, Y. Wei, and X. Zhang, "Consistent Approximation of Fractional Order Operators," *Journal of Dynamic Systems, Measurement, and Control*, vol. 143, no. 8, pp. 1–12, 2021, doi: 10.1115/1.4050393.

[31] S. Narayan, U. Mehta, R. Iro, H. Sıkwa'ae, K. Kothari, and N. Singh, "Realization of fractional band pass filter on reconfigurable analog device," *International Review of Applied Sciences and Engineering*, vol. 13, no. 1, pp. 63–69, 2021, doi: 10.1556/1848.2021.00308.

[32] A. N. Khovanskiĭ, *The application of continued fractions and their generalizations to problems in approximation theory*. Groningen, Netherlands: Noordhoff, 1963, doi: 10.2307/2003784.

[33] M. E. V. Valkenburg, *Introduction to Modern Network Synthesis*. United States: Wiley International, 1960.

[34] M. A. Siddiqi, *Continuous time active analog filters*. Cambridge, UK: Cambridge University Press, 2019, doi: 10.1017/9781108762632.

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