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Advanced trajectory tracking control for wheeled mobile robots under actuator faults and slippage

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ABSTRACT

Trajectory tracking control for wheeled mobile robots (WMRs) faces significant challenges in real-world applications due to actuator faults, longitudinal and lateral slippage. This study proposes an innovative dual-loop control structure combining adaptive sliding mode control (ASMC) and backstepping control (BC), supported by robust fault observers, to address these challenges. The dynamic loop employs ASMC to handle model uncertainties and disturbances, while the kinematic loop integrates BC with fault information provided by the observers, enabling real-time error compensation. Simulation results show that the proposed method significantly reduces tracking errors and improves stabilization time compared to traditional SMC and ASMC controllers. The system exhibits enhanced fault tolerance and disturbance rejection, maintaining stability under both normal and faulty conditions. The effectiveness of this approach is demonstrated through simulations and theoretical analysis, ensuring system stability using Lyapunov stability theory. The proposed method enhances robustness, adaptability, and stability of WMRs, contributing significantly to the field of mobile robotics under adverse conditions.

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1. INTRODUCTION

Wheeled mobile robots (WMRs) are crucial in fields like manufacturing, logistics, healthcare, and autonomous vehicles due to their flexibility and efficiency. Accurate trajectory tracking is essential for WMR success, but they face challenges such as actuator faults, slippage, and environmental disturbances, necessitating robust control strategies. With technological advances, various control methods have been developed to address these challenges in WMR systems.

One such strategy is sliding mode control (SMC), widely used to enhance stability and reject disturbances in WMRs. In study [1], the authors developed a super-twisted fractional SMC with an fault observer, which reduces chattering and improves robustness through real-time fault compensation. The work in [2] proposed discrete-time SMC for trajectory tracking on the PowerBot, enhancing practical tracking performance. Meanwhile, Yang and Pan [3] applied SMC to mitigate active noise, increasing convergence speed and tracking accuracy. In addition to SMC, adaptive control effectively handles uncertainties in WMR systems. Gao *et al*. [4] focused on adaptive control for unknown slippage, using neural networks for real-time weight tuning to minimize tracking errors. Similarly, Shojaei *et al*. [5] designed an adaptive controller to handle parametric uncertainties in nonholonomic WMRs using a nonlinear control law. The paper [6] further extended this by developing a modular adaptive controller that separates control and update laws, enhancing both flexibility and transient response. Observer-based control is another essential approach for estimating and compensating unknown parameters and disturbances. Qin *et al*. [7] utilized an enhanced extended state observer (EESO) combined with a fixed-time tracking controller to maintain system stability. The study [8] integrated disturbance observers to reject approximation errors, while Ding *et al.* [9] developed an adaptive fuzzy control method that ensures prescribed trajectory tracking performance by combining an observer with a fuzzy logic controller to handle external disturbances and system uncertainties. Cui [10] further proposed an adaptive observer-based approach to manage unknown longitudinal slippage using the Lyapunov method, ensuring stable tracking with nonlinear feedback. Finite-time control methods have also been explored for WMRs to improve convergence speed. Zhang *et al*. [11] introduced a finite-time trajectory tracking approach that ensures global progressive stability, while Moudoud *et al*. [12] proposed an adaptive terminal integral sliding mode control (ATISMC) for trajectory tracking, dynamically adjusting gains for robustness and reducing chattering. Robust control techniques further enhance system stability, particularly in the presence of disturbances. Ma and Tso [13] developed a robust exponential regulation technique that ensures exponential convergence despite parameter uncertainties, while Yang *et al*. [14] used an extended state observer (ESO) to estimate unknown disturbances, facilitating accurate tracking and disturbance compensation. The work in [15] created a robust tracking scheme that handles skidding, slipping, and input disturbances, with experimental validation demonstrating its effectiveness. Zhao *et al*. [16] designed an adaptive neural backstepping control system to ensure stability through Filippov theory and Lyapunov criteria. Fierro and Lewis [17] explored backstepping for kinematic and dynamic control of nonholonomic WMRs, guaranteeing asymptotic stability. Xia *et al*. [18] integrated SMC with backstepping, using ESO to ensure that the system's state variables converge to the desired reference. Hybrid control methods, which combine various approaches, offer further optimization of WMR performance. Koubaa *et al*. [19] proposed an ASMC method for nonholonomic WMRs, addressing kinematic and dynamic uncertainties while compensating for disturbances. Wang *et al*. [20] implemented ESO-based ASMC to improve trajectory tracking under unknown sliding and slipping conditions, achieving higher tracking accuracy and better disturbance rejection than traditional ASMC.

Despite these advances, significant gaps remain in controlling WMRs under both slippage and actuator faults. For instance, Wang and Zhai [21] address slippage without considering actuator faults, while [22], [23] focus on actuator faults but neglect slippage. Unified observers for both issues, such as those used by Cui [10] and Vu *et al*. [24], introduce complexity and challenge system stability. Additionally, some studies, like those by Taherisarteshnizi and Alipour [25], focus solely on slippage, while others, such as Hao and Zhou [26], handle actuator faults and uncertainties without addressing slippage.

This paper aims to bridge these gaps by developing an advanced adaptive control method that integrates observer and disturbance rejection techniques, effectively addressing both slippage and actuator faults simultaneously. The proposed solution employs a dual-loop control structure that combines ASMC and BC, supported by robust fault observers. The dynamic control loop uses ASMC to manage model uncertainties and disturbances, while the kinematic control loop integrates BC with real-time fault information from the observers. This integrated approach not only allows the system to respond quickly to disturbances but also detects and compensates for faults in a timely manner. The result is enhanced robustness, adaptability, and stability for WMRs, significantly improving performance under real-world conditions involving both actuator faults and slippage. The effectiveness of this method is demonstrated through comprehensive simulations and theoretical analysis, ensuring system stability using Lyapunov stability theory.

The main contributions of this paper include: i) considering the aspects of the actuator fault and robot slippage, a dual-loop control structure combining ASMC and BC for WMRs is developed; ii) the actuator faults observation and disturbance observation are separated to ensure the effectiveness of the proposed controller, and the effects of actuator faults and disturbance are synthesized for real-time compensation; and iii) extensive simulations and theoretical analysis demonstrating the method's effectiveness and stability.

The paper is organized as follows: section 2 reviews related work on control strategies for managing disturbances, slippage, and actuator faults in WMRs. Section 3 describes the proposed control method, detailing the implementation of ASMC and backstepping control with fault observers. Provides the mathematical framework, including the derivation of control laws and stability analysis. Section 4 demonstrates the effectiveness of the proposed method through various simulation scenarios. Finally, section 5 concludes the paper and suggesting directions for future research.

2. MATHEMATICAL MODEL OF NWMR

The non-holonomic robot model with two active differential wheels that we developed has the structure shown in Figure 1 with the basic parameters of the robot including r, $2L$ which are the wheel radius and the distance between the two active wheel, respectively, m is the mass of the robot, theta is the steering angle of the robot, the angular velocities of the left and right wheels are $\omega_R = \dot{\varphi}_R$ and $\omega_L = \dot{\varphi}_L$ respectively. In this design, the coordinates of the robot's center of mass is point G, d is distance between of O_B and G. v

and ω denote the linear velocity and angular velocity of the robot. Hypothesized sliding components include δ_{sl} denotes the lateral skidding velocity, ϖ_R and ϖ_L present the perturbed angular velocity of right and left wheels, σ_v is the longitudinal slipping velocity, σ_ω is the perturbed angular velocity of WMR be effected wheel slipping, and $\sigma_v = \frac{r(\varpi_R+\varpi_L)}{2}$; $\sigma_\omega = \frac{r(\varpi_R+\varpi_L)}{2h}$.

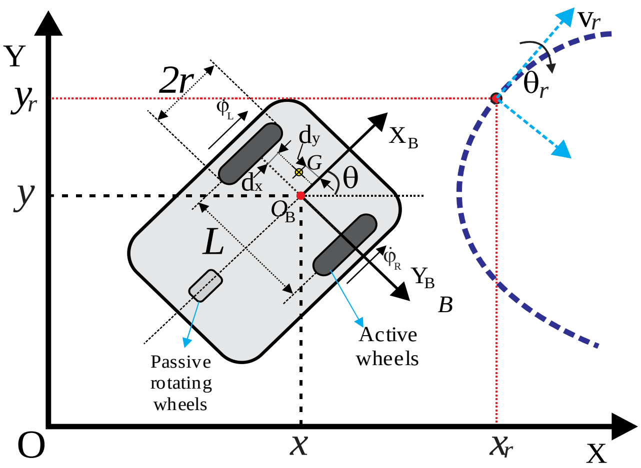


Figure 1. NWMR's structure

In this study, denote $C\theta = cos\,\theta$, $S\theta = sin\,\theta$, the non-holonomic constraints of WMR be expressed as (1):

$$\begin{cases} -\dot{x}S\theta + \dot{y}C\theta - d\dot{\theta} = \delta_{sl} \\ \dot{x}C\theta + \dot{y}S\theta + h\dot{\theta} = r(\dot{\varphi}_R - \varpi_R) \\ \dot{x}C\theta + \dot{y}S\theta - h\dot{\theta} = r(\dot{\varphi}_R - \varpi_L) \end{cases} \quad (1)$$

Presented with matrix form, (1) become:

$$A(q)\dot{q} = \Delta_S \quad (2)$$

where, $A(q) = \begin{bmatrix} -S\theta & C\theta & -d & 0 & 0 \\ C\theta & C\theta & h & -r & 0 \\ C\theta & S\theta & -h & 0 & -r \end{bmatrix}$ is the non-holonomic constraint matrix, $q = [x \quad y \quad \theta \quad \varphi_R \quad \varphi_L]^T$, $\Delta_S = [\delta_{sl} \quad -r\varpi_R \quad -r\varpi_L]^T$ caused by wheel slip.

A matrix $S(q) = \begin{bmatrix} C\theta & S\theta & 0 & \frac{1}{r} & \frac{1}{r} \\ -dS\theta & dC\theta & 1 & \frac{h}{r} & \frac{-h}{r} \end{bmatrix}^T$ is defined to satisfy $A(q)S(q) = 0$.

The kinematics of WMR can be shown as (3):

$$\dot{q} = S(q)(\eta - \sigma) + U_D \quad (3)$$

where, $\eta = [v \quad \omega]^T$,$\sigma = [\sigma_v \quad \sigma_\omega]^T$, U_D denotes the unmatched disturbance, matrix due to the perturbed non-holonomic constraint, and:

$$U_D = \begin{bmatrix} -S\theta & 0 & 0 \\ C\theta & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{sl} \\ \varpi_R \\ \varpi_L \end{bmatrix} = [-S\theta\delta_{sl} \quad C\theta\delta_{sl} \quad 0 \quad \varpi_R \quad \varpi_L]^T$$

With the assumption as the perturbation σ, δ_{sl} are bounded. The derivation of (3), we obtained:

$$\ddot{q} = \dot{S}(q)(\eta - \sigma) + S(q)(\dot{\eta} - \dot{\sigma}) + \dot{U}_D \quad (4)$$

According to [4], [5], [15], [16], in the Lagrange form, the dynamic equation of WMR can be presented as (5):

$$M_0(q)\ddot{q} + C_0(q,\dot{q})\dot{q} + G_0(q) + \tau_d = B_0(q)\tau - A^T(q)\lambda \tag{5}$$

In this case, $M_0(q) \in R^{5x5}$ is a symmetric, positive definite inertia matrix, $C_0(q,\dot{q}) \in R^{5x5}$ is the centripetal and Coriolis matrix, $G_0(q) \in R^{5x1}$ is the gravitational vector ($G_0(q) = 0$), $\tau_d \in R^{5x1}$ denotes the bounded external disturbances, $B_0(q) \in R^{5x2}$ is the input transformation matrix, $\tau \in R^{2x1}$ is the input torque, λ is the vector of constraint force.

$$M(q) = \begin{bmatrix} m_{11} & 0 & m_{13} & 0 & 0 \\ 0 & m_{22} & m_{23} & 0 & 0 \\ m_{31} & m_{32} & m_{33} & 0 & 0 \\ 0 & 0 & 0 & m_{44} & 0 \\ 0 & 0 & 0 & 0 & m_{55} \end{bmatrix}; C(q,\dot{q}) = \begin{bmatrix} 0 & 0 & c_{13} & 0 & 0 \\ 0 & 0 & c_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}; B(q) = \begin{bmatrix} C\theta & C\theta \\ S\theta & S\theta \\ 2h & -2h \\ r & 0 \\ 0 & r \end{bmatrix}$$

herein:
$m_{11} = m_{22} = m;$
$m_{13} = m_{31} = -mdS\theta;$
$m_{23} = m_{32} = mdC\theta;$
$m_{33} = I;$
$m_{44} = m_{55} = I_W;$
$c_{13} = -md\omega C\theta;$
$c_{23} = -md\omega S\theta.$
Substituting (4) into (5), then multiplying both sides of the resulting expression by $S^T(q)$ we have:

$$M_1(\dot{\eta} - \dot{\sigma}) + N_1(\eta - \sigma) + P_1 + S^T(q)\tau_d = S^T(q)B_0(q)\tau \tag{1}$$

where,
$M_1 = S^T(q)M_0(q)S(q);$
$P_1 = S^T(q)M_0(q)\dot{U}_D + S^T(q)C_0(q,\dot{q})U_D;$
$B_1 = S^T(q)B_0(q);$
$N_1 = S^T(q)M_0(q)\dot{S}(q) + S^T(q)C_0(q,\dot{q})S(q).$
The dynamic equation of WMR be inferred:

$$\dot{\eta} = M\eta + N\tau + D \tag{7}$$

where, $M = -{M_1}^{-1}N_1;$ $N = {M_1}^{-1}B_1;$ $D = D_{pa} + {M_1}^{-1}N_1\sigma + \dot{\sigma} - {M_1}^{-1}(P_1 + S^T(q)\tau_d);$ $D_{pa} = \Delta M\eta + \Delta N\tau$; $\Delta M, \Delta N$ are parameter uncertainty.

3. DESIGN CONTROLLER

In this section, the observer-based advanced controller is designed to achieve the highest tracking performance for NWMR. Firstly, the trajectory tracking controller for NWMR is established on the basic of the backstepping technique and the ESO. Then, the dynamic controller is constructed on the basic of the ASMC as shown in Figure 2. Finally, the system stability of this loop is verified by Lyapunove theorem.

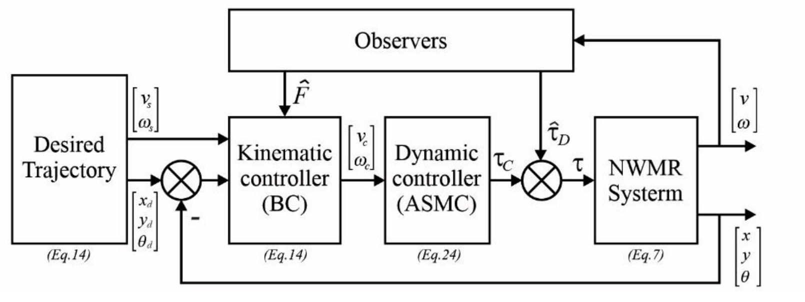


Figure 2. The structure of proposed controller

3.1. Trajectory tracking controller

With two coordinates system in Figure 1. The tracking error vector of the NWMR is expressed as (8):

$$e = \begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} C\theta & S\theta & 0 \\ -S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_s - x \\ x_s - x \\ x_s - x \end{bmatrix} \tag{8}$$

The derivation of (8), we obtained:

$$\dot{e} = \begin{bmatrix} -1 & e_y \\ 0 & -e_x \\ 0 & -1 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix} + \begin{bmatrix} v_s C e_\theta \\ v_s S e_\theta \\ \omega_s \end{bmatrix} \tag{9}$$

where v_s, ω_s are linear and angular velocity of the virtual vehicle, respectively.

The actuator fault of the WMR be consider in this paper. For some reason, one or two motors have a problem, affecting the robot's performance, changing the robot's actual linear speed and angular speed.

$$\begin{cases} v_r = v - \Delta v \\ \omega_r = \omega - \Delta\omega \end{cases} \tag{10}$$

where $\Delta v = \varepsilon_1 v, \Delta\omega = \varepsilon_2 \omega$ $(0 < \varepsilon_i < 1)$ are bound, so $\dot{\Delta v} = 0; \dot{\Delta\omega} = 0$.
In this case, when considering the faults (9), (10) be presented as (11):

$$\dot{e} = \begin{bmatrix} \omega_s e_y \\ v_s S e_\theta - e_x \omega_s \\ 0 \end{bmatrix} + \begin{bmatrix} 1 & -e_y \\ 0 & e_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -v + v_s C e_\theta \\ \omega_s - \omega \end{bmatrix} + \begin{bmatrix} 1 & -e_y \\ 0 & e_x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta v \\ \Delta\omega \end{bmatrix} \tag{11}$$

Set:

$$f(e) = [\omega_s e_y \quad v_s S e_\theta - e_x \omega_s \quad 0]^T;\ F = [\Delta v \quad \Delta v]^T;\ u = [v - v_s C e_\theta \quad \omega_s - \omega]^T;$$

$$g_1(e) = \begin{bmatrix} -1 & 0 & 0 \\ e_y & e_x & 1 \end{bmatrix}^T;\ g_2(e) = \begin{bmatrix} 1 & 0 & 0 \\ -e_y & e_x & 1 \end{bmatrix}^T$$

Can be represented with form:

$$\begin{cases} \dot{e} = f(e) + g_1(e)u + g_2(e)F \\ \eta = h(e) \end{cases} \tag{12}$$

According to Kaldmäe and Kotta [27], the system state *e* and input *u* are known, and $\dot{F} = 0$. The fault F can be estimate via an observer, which have structure showed as (13).

$$\begin{cases} \dot{z} = -L(e))[g_2(e)z + g_2(e)p(e) + f(e) + g_1(e)u] \\ \hat{F} = z + p(e) \end{cases} \tag{13}$$

where $p(e)$ and $L(e)$ are the observer gains, which satisfying $L(e) = \frac{\partial p(e)}{\partial e}$. According to this study [27] $e_F = F - \hat{F}$ to be stable to zero.

By the observer model (13), the actuator faults $\hat{F} = [\Delta\hat{v} \quad \Delta\hat{\omega}]^T$ be estimated. The law control for trajectory tracking be proposed as (14):

$$\eta_c = \begin{bmatrix} v_c \\ \omega_c \end{bmatrix} = \begin{bmatrix} v_s C e_\theta + k_x e_x + 2 e_y \Delta\hat{\omega} + \Delta\hat{v} \\ \omega_s + k_y v_s e_y + k_{th} S e_\theta + \Delta\hat{\omega} \end{bmatrix} \tag{14}$$

where k_x,k_y,$k_{th} > 0$ are design parameters.
The Lyapunov function candidate is selected as (15):

$$V_K = \frac{1 - \cos e_\theta}{k_y} + \frac{e_x^2 + e_y^2}{2} \tag{15}$$

Derivation of (15) and combine with (14) and (9), we obtained:

$$\dot{V}_K = e_x\dot{e}_x + e_y\dot{e}_y + \frac{\dot{e}_\theta Se_\theta}{k_y} = e_x\left(-v + v_s Ce_\theta + 2\Delta\omega e_y + \Delta v\right) + \frac{Se_\theta(\omega_s - \omega + k_y e_y v_s + \Delta\omega)}{k_y}$$
$$= -k_x e_x^2 - \frac{k_{th}}{k_y} Se_\theta{}^2 \leq 0 \tag{16}$$

The system to be stable according to the Lyapunov stability theorem [28].

3.2. Dynamic controller

The total unknown total uncertainty disturbance D in (7) have not been determined. To minimize the impact of this total disturbance, an ESO is proposed for the online estimation of internal and external disturbances [29]. By the let $x_1 = \eta$; $x_2 = \dot{\eta}$; $x_3 = D$, (7) can be presented as (17):

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = N\tau + f(x_1) + x_3 \\ \dot{x}_3 = \xi \end{cases} \tag{17}$$

To estimate of the unknown disturbance, an ESO with third-order was proposed as (18):

$$\begin{cases} \dot{\hat{x}}_3 = -\lambda_3 fal(e_{x_1}, \zeta, \alpha) \\ \dot{\hat{x}}_2 = N\tau - \lambda_2 fal(e_{x_1}, \zeta, \alpha) + \hat{x}_3 \\ \dot{\hat{x}}_1 = \hat{x}_2 - \lambda_1 e_{x_1} \end{cases} \tag{18}$$

where $e_{x_1} = x_1 - \hat{x}_1$ is estimate errors of x_1, and $\hat{x}_1\hat{x}_2\hat{x}_3$ denotes the estimation of η , $\dot{\eta}$, D, respectively. $0 < \alpha < 1, \zeta > 0$, $fal(.)$ is the nonlinear function [29].

$$fal(e_{x_1}, \zeta, \alpha) = \begin{cases} \frac{e_{x_1}}{\zeta^{1-\alpha}} & \text{if } |e_{x_1}| \leq \zeta \\ |e_{x_1}|^\alpha sign(e_{x_1}) & \text{if } |e_{x_1}| > \zeta \end{cases} \tag{19}$$

Based on the estimate of the total disturbance with estimate errors ($e_D = D - \hat{x}_3$) converges to zero from the observer [29], the integral sliding controller is used to further improve the turbulence compensation ability as well as improve the orbit tracking performance of the robot.
Denote $e_\eta = \eta_c - \eta$, the sliding surface is designed as (20):

$$s = e_\eta + k_s \int_0^t e_\eta dt \tag{20}$$

The derivation of (20), then combine with (4) we have:

$$\dot{s} = (\dot{\eta}_c - \dot{\eta}) + k_s e_\eta = (\dot{\eta}_c - M\eta - N\tau - D) + k_s e_\eta \tag{21}$$

Order to the state trajectory to remain on the sliding surface, so $\dot{s} = 0$ is a necessary condition. The equivalent control law can be obtained:

$$u_{eq} = N^{-1}(\dot{\eta}_c - M\eta - \hat{x}_3 + k_s e_\eta) = N^{-1}(\dot{\eta}_c - M\eta - \hat{x}_3 + k_s e_\eta) \tag{22}$$

The discontinuous control law should be added to maintain the sliding condition under effective by uncertainties and disturbances always exist in real systems. Hence, the law of controller is presented as (23):

$$\tau = u_{SMC} = u_{eq} + u_{sw}, u_{sw} = N^{-1}\hat{\rho} sign(s) \tag{23}$$

In this law, we used switching adaptive gain $\hat{\rho}$ be estimation choose to compensate for the system uncertainties and disturbances. To reduce chattering, the conventional $sign$ function be replaced by the continuous $tanh$ function, which approximates the $sign$ function.
Denote $e_\rho = \rho - \hat{\rho}$, the final control law for SMC-PI based on ESO can be proposed as (24):

$$\begin{cases} \tau = u_{SMC} = N^{-1}(\dot{\eta}_c - M\eta - \hat{x}_3 + k_s e_\eta + \hat{\rho}\, tanh(s)) \\ \dot{\hat{\rho}} = \rho|s|\left(1 + \frac{2}{e_\rho \pi}\left|atan(\frac{\xi}{s})\right|\right) \end{cases} ,\xi \text{ is a small positive constant} \tag{24}$$

The Lyapunov function candidate is selected as (25):

$$V_D = \frac{1}{2} s^T s + \frac{1}{2\rho} e_\rho^2 \tag{25}$$

The derivation of (25), then combine with (24) we have:

$$\begin{aligned}\dot{V}_D &= s^T \left((\dot{\eta}_c - M\eta - N\tau - D) + k_s e_\eta \right) - \frac{1}{\rho} e_\rho \dot{\hat{\rho}} = -s^T (D - \hat{x}_3) - s^T \hat{\rho}\, tanh(s) - \frac{1}{\rho} e_\rho \dot{\hat{\rho}} \\ &= -s^T (\hat{\rho}\, tanh(s) + e_D) - \frac{1}{\rho} e_\rho \dot{\hat{\rho}} \leq -|s|(\rho + e_D) - \frac{2|s|}{\pi} \left| atan\left(\frac{\xi}{s}\right) \right| \leq -\frac{2|s|}{\pi} \left| atan\left(\frac{\xi}{s}\right) \right| \leq 0\end{aligned} \tag{26}$$

So, the system (17) with ESO (18) and adaptive control law (24) to be stable according to the Lyapunov stability theorem [28]. The full structure of the proposed controller for the NWMR system be presented in the Figure 2.

4. RESULTS SIMULATION AND DISCUSSION

This section presents the key findings of the study. The simulation results are provided for two main scenarios: i) scenarios with disturbances and wheel slippage but without actuator faults and ii) scenarios with disturbances, wheel slippage, and actuator faults. The performance of the proposed controller (combining ASMC, backstepping, and fault observers) is compared with SMC and ASMC controllers without fault observers. The parameters of WMR and controller are show in Tables 1 and 2.

Table 1. The WMR's parameters

Parameters	Detail	Value
m (Kg)	Total mass of the robot's frame	15
r (m)	The radius of the wheels	0.1
I (Kgm^2)	Total moment of inertia	2.5
I_W (Kgm^2)	Moment of inertia of driving wheel	0.001
$2h$ (m)	The distance between the two driving wheels	0.4
d (m)	The distance between the point O_B and G	0.05

Table 2. Parameters of controllers for simulation

Controller	Parameters
SMC without observer	$k_x = 1.2, k_y = 250, k_{th} = 5, k_s = 5$
ASMC without observer	$k_x = 1.2, k_y = 250, k_{th} = 5, k_s = 5, \rho = 0.1, \xi = 0.01,$
Proposed	$k_x = 1.2, k_y = 250, k_{th} = 5, k_s = 50, \rho = 0.1, \xi = 0.01, \alpha = 0.01, \zeta = 0.1, \lambda_1 = 10, \lambda_2 = 20, \lambda_3 = 30$

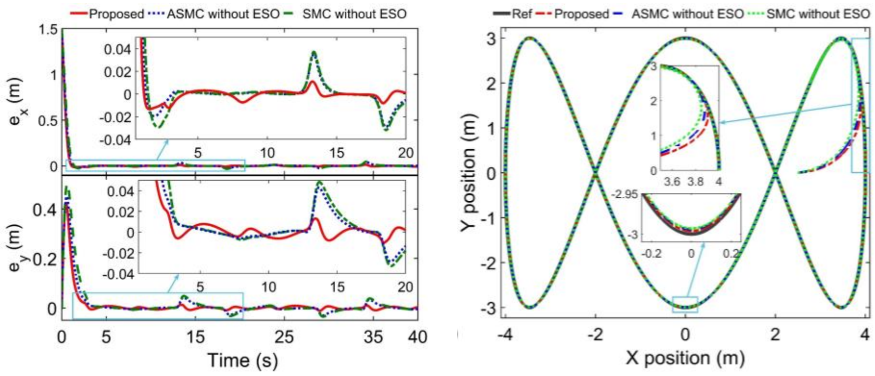
Desired trajectory is selected following as: $q_d = [4\,cos(0.2t) \quad 3\,cos(0.6t) \quad 0.2t]^T$, initial state: $q_0 = [2.5 \quad 0 \quad 0]^T$. The affection of total lumped and faults at 10th second and finish at 35th second. Total time for simulation is 40 seconds.

Affection of slipping show as: $\Delta_S = [\delta_{sl} \quad -r\varpi_R \quad -r\varpi_L]^T$, where $\delta_{sl} = 2 + 0.8sin(t)$, $\varpi_R = \beta\, sin(t)$, $\varpi_L = \beta\, cos(t)$ with $0 < \beta < 0.2$ is a random number. Fault effective on actuator presented by: $\Delta v = \varepsilon v$, $\Delta\omega = \varepsilon\omega$, ε is a random number, which have value limit by $0.125 < \varepsilon < 0.25$.

4.1. The simulation with total lumped effective without faults

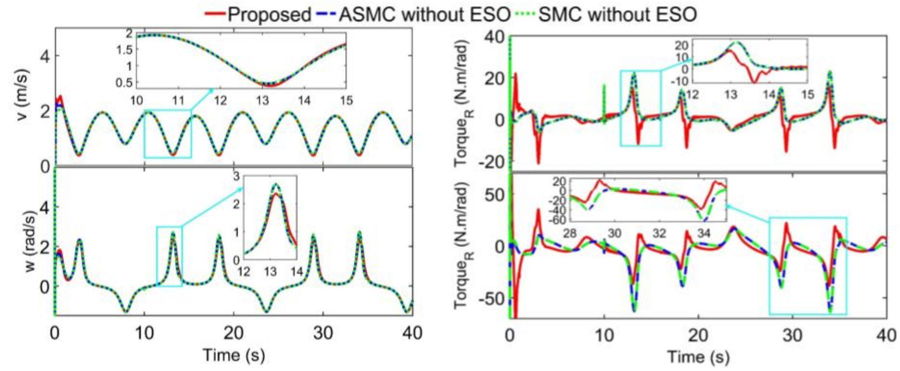
As shown in Figure 3, which presents the position tracking analysis in Case 1, Figure 3(a) illustrates the tracking errors along the X and Y axes, while Figure 3(b) compares the achieved trajectories in the XY plane. However, the proposed controller showed significantly smaller tracking errors compared to SMC and ASMC without observers. The tracking errors along the x and y axes highlight that the proposed controller achieves faster stabilization, especially at sharp turns. This result indicates the superior performance of the proposed controller in terms of trajectory tracking due to its capability to compensate for disturbances and adjust parameters in real-time using fault observer information.

Additionally, the results regarding linear and angular velocities, as well as the torque applied to the wheels (see Figures 4(a) and (b)), show that the proposed controller operates more stably with fewer fluctuations than the other two controllers. This indicates better disturbance rejection and more accurate trajectory tracking in dynamic environments.



(a) (b)

Figure 3. Position tracking analysis in Case 1: (a) tracking errors along the X and Y axes and (b) comparison of achieved trajectories in the XY plane

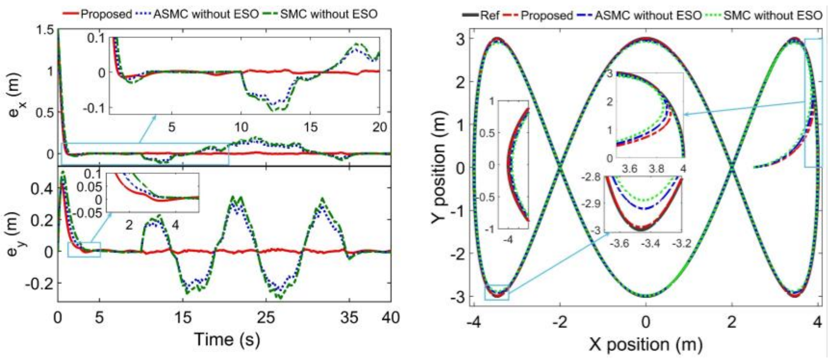


(a) (b)

Figure 4. Actual velocities and input torques at the wheels in Case 1; (a) velocity and (b) torque

4.2. The simulation with total lumped and faults

In scenarios where actuator faults were introduced, the differences between the controllers became more pronounced. Figure 5 presents the position tracking analysis under actuator faults, where Figure 5(a) illustrates the tracking errors along the X and Y axes, and Figure 5(b) compares the achieved trajectories in the XY plane.



(a) (b)

Figure 5. Position tracking analysis in Case 2; (a) tracking errors along the X and Y axes and (b) comparison of achieved trajectories in the XY plane

The results indicate that, even with actuator faults (Figure 6), the proposed controller maintained the smallest and most stable tracking errors, while the SMC and ASMC controllers without observers exhibited larger errors and took longer to reach a steady state. This finding is further supported by Figure 7, which details the actual velocities and input torques at the wheels in Case 2. Specifically, Figure 7(a) displays the velocity profiles, while Figure 7(b) depicts the corresponding torques applied to the wheels. These results confirm that combining ASMC, backstepping, and fault observers enhances the system's adaptability to actuator faults. The fault observer provides real-time fault estimation, allowing the controller to adjust effectively, thereby minimizing the impact of faults and improving trajectory tracking accuracy.

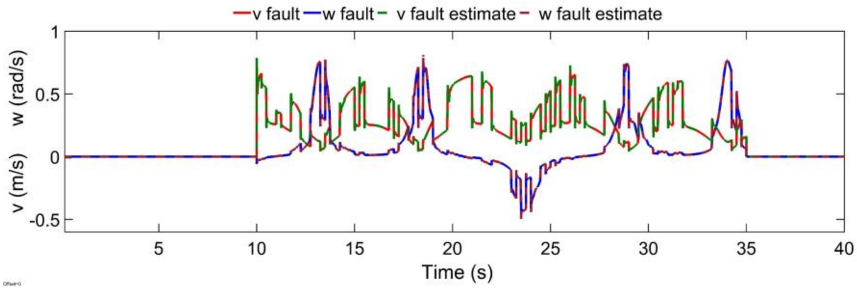
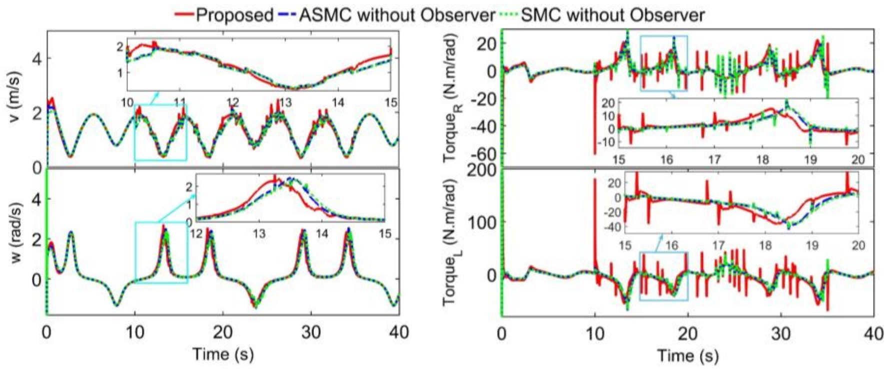


Figure 6. The faults and estimate



(a) (b)

Figure 7. Actual velocities and input torques at the wheels in Case 2; (a) velocity and (b) torque

Our results align with previous research on applying sliding mode and backstepping control methods to WMRs. In the paper [21], the authors focused on managing wheel slippage but did not address actuator faults. Jin *et al.* [22] dealt with actuator faults but overlooked slippage. Unlike these studies, our proposed method successfully handles both issues simultaneously by integrating ASMC, backstepping, and fault observers, allowing for real-time fault detection and compensation, which leads to improved robustness and accuracy.

Recent studies, such as the paper [24], explored robust adaptive control for WMRs in environments with disturbances and wheel slips. Their approach, while effective in managing wheel slips, does not fully address actuator faults, which our study integrates into a unified control strategy. Similarly, the article [20] proposed an adaptive sliding mode control (ASMC) for trajectory tracking considering skidding and slipping, but their method does not incorporate actuator fault observers, which is a key strength of our approach.

Additionally, the work in [30] demonstrated an adaptive fault-tolerant control method that addresses actuator faults but does not handle wheel slippage. Our research builds upon these advancements by integrating a fault observer that effectively compensates for both actuator faults and slippage, providing a more comprehensive solution for real-world WMR applications. Compared to other recent works such as the authors in [25], which focused on fuzzy dynamic control for lateral and longitudinal wheel slippage, our approach enhances the control structure by incorporating both ASMC and backstepping, supported by fault observers, making it more robust in handling both slippage and actuator faults simultaneously. This study, therefore contributes to closing the gap in research where both wheel slippage and actuator faults need to be addressed concurrently. The proposed dual-loop control structure provides a novel, comprehensive solution that enhances the adaptability and stability of WMRs under adverse conditions.

While the simulation results demonstrate the superior performance of the proposed controller, there are some limitations to this study. The simulations were conducted in an ideal environment with specific types of disturbances and faults. In real-world applications, more complex types of disturbances and faults may occur, which require further verification through practical experiments. Additionally, the impact of sensor faults has not been fully considered in the simulations.

5. CONCLUSION

This study has demonstrated that combining ASMC and BC with fault observers is an effective solution for trajectory tracking control of WMR, particularly in environments with disturbances and actuator faults. The proposed controller has shown superior performance in compensating for disturbances and minimizing tracking errors compared to SMC and ASMC controllers without fault observers. A key contribution of this study is the simultaneous handling of both disturbances and actuator faults, which previous studies have not fully addressed. This approach not only improves system stability and tracking accuracy but also opens new avenues for developing adaptive control systems that can respond to complex and dynamic environments. The findings of this study are not only significant from a theoretical perspective but also have the potential for widespread practical applications in real-world robotic systems. Future research should focus on conducting practical experiments to further verify and refine the proposed method, particularly in more complex environments such as rough terrains or industrial applications. The main question that arises for future work is how to integrate more advanced control methods, such as fuzzy systems or deep learning, to further enhance the system's ability to handle real-world challenges. These findings are expected to have a substantial impact on the field of autonomous mobile robotics, helping improve performance and reliability in a wide range of applications.

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C : **C**onceptualization
M : **M**ethodology
So : **So**ftware
Va : **Va**lidation
Fo : **Fo**rmal analysis
I : **I**nvestigation
R : **R**esources
D : **D**ata Curation
O : Writing - **O**riginal Draft
E : Writing - Review & **E**diting
Vi : **Vi**sualization
Su : **Su**pervision
P : **P**roject administration
Fu : **Fu**nding acquisition

CONFLICT OF INTEREST STATEMENT

Authors state no conflict of interest.

INFORMED CONSENT

We have obtained informed consent from all individuals included in this study.

DATA AVAILABILITY

Data availability is not applicable to this paper as no new data were created or analyzed in this study.

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