

Realization of Fractional order Lowpass Filter using different approximation Techniques

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ABSTRACT

Many research groups are starting to pay serious attention to the problem of fractional-order circuits. In this paper, a new approach to designing low-pass filters of fractional order (FOLPF) is presented. Finding a rational approximation of the fractional laplace operator s^α is a crucial step in the design of fractional order filters. A comparative study of the most widely used approximation techniques named, Continued Fraction Expansion method, R.Elkhazali method is performed. Then the transfer function of the proposed FOLPF is calculated. Then, using operational amplifier the proposed filter is synthesized. The proposed circuit is simulated using Texas instruments TINA Software. The results obtained outperform the existing methods.

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1. INTRODUCTION

The simulation of the physical environment can make use of an extension of classic integral and differential equations known as fractional order differential and integral equations [1]. The fractional differential equations could not be solved using any of the methods that are available, so the integer-order models were used instead. Several different approaches are now being utilised in the process of developing applications predicated on fractional derivatives and integration. It is possible to improve the characterization of a wide variety of dynamic systems by employing non-integer order dynamic models that are founded on fractional calculus, differentiation, or integration. Calculus in its classical form is built on the foundation of differentiation and integration of integer orders [1]. Fractional order systems makes use of fractional order differentiation and integration as constituent elements. Recent applications of fractional order systems in electrical engineering is an appreciable thing. A fractional order element (also called as fractance device) is designated with s^α .

Realization of fractance device is an important step for the design of fractional order filters. A continued fraction expansion based realization of fractance device is discussed in [2]. A fractional impedance of order $1/2$ is realized using Hilberts structure and is used in the design of fractional order controllers [3]. Fractional order filters are used in many of the electronic systems. Fractional order Low pass, High pass, Band pass, Band reject and all pass filters of first and second order are very well designed in the literature. They are designed in both current mode and voltage mode. Operational amplifiers (Op Amps), differential difference current conveyors (DDCCs), current feedback operational amplifiers (CFOAs), second generation current conveyors (CCIIIs), Operational transconductance amplifiers (OTAs), or current amplifiers (CAs) are just a few examples

of the many commercially available active elements that can be used to design a fractional-order filter[4]. One can use a variety of topologies to create the transfer function that resembles the fractional-order filter. The topologies FLF (Follow the Leader Feedback), IFLF (Inverse Follow the Leader Feedback), or SAB (Single Amp Biquad) in conjunction with a 1st-order filter are the most often utilised topologies. Even though many topologies exist, it is the design requirement of the end user to use the particular topology.

In [5] the authors have discussed the design and electronic implementation of the fractional-order controller and controlled system. This system is based on the Laplace transfer function of the corresponding electronic circuit being equivalent to the Laplace transfer function of the original Fractional Order systems. Authors goal in [6] is to synthesise a fractance circuit in an electronic circuit to achieve fractional order derivatives. Two different types of fractances were synthesised and linked in an operational amplifier as passive RC circuits to achieve the realisation. A.Iqbal et.al in [7] have studied about different types of approximations of fractional order systems. Realization of fractional order filters at microwave region is studied in [8]. P.K.Tewari et.al have designed and performed the PSPICE based simulation of Operational Transresistance Amplifier based Fractional Low pass Filter of order 1.5[9]. Resistor less realization of generalized filters of fractional order using Operational Transconductance Amplifiers as active elements is realized in [10]. Operational amplifier based design of fractional order controllers is studied using simulation tools in [11] by A.K.Mahmood et.al. The study in [12] uses CADENCE-PSPICE to build and simulate simple operational transconductance amplifier (OTA)-based fractional order low-pass and high-pass filters of order $(n + \alpha)$, where $0 < \alpha < 1$ and $n \geq 1$. For any order $0 < \alpha < 1$, the fractional-order low-pass filter based on a single fractance element is represented in [13] by using optimal integer-order transfer function approximations. Initially, integer-order filter coefficients for the FLF are found directly in the range of 0.01 to 0.99 with a step size of 0.01. This is done by employing the colliding bodies optimisation technique. A novel rational approximation for the fractional order operator is proposed in [14]. Signal flow graphs synthesise OTA based fractional element in[15]. Theory, simulation, and approximation function match. Genetic algorithm is used in the synthesis and optimization of fractional order elements[16]. Different passive realization of fractional order capacitors is studied extensively in [17].

Design of the FOLPF necessitates the approximation of the operator to a numerical degree. These approximation techniques are proposed by the researchers in different instants of time. The work done in [18] introduces fractional order low-pass, high-pass, band-pass, and band-reject filter responses at the same circuit using low-voltage active elements based on a general filter topology. Using 0.35um TSMC CMOS technology characteristics and SPICE software, the designed circuits are simulated. In this paper, a comparative study of the available rational approximation techniques is carried out. Using Laplace operators of various fractional orders, this paper[19] presents a straightforward method for creating fractional-order filters with transfer functions [19]. S.K.Mitra et.al has proposed a fractional order butterworth lowpass filter using Differential Voltage Current Conveyor[20]. A demonstrated example of the realization of fractional order element using partial fraction expansion method is discussed in [21,22]. In order to test and verify electrical bio-impedance circuits and systems, his work contrasts two design techniques by simulating skin tissue and AgCl electrodes[23]. The models represent bio-impedance characteristics up to 10 kHz, are implemented with active components, and are based on fractional-order elements. Work in [24] analyses a fractional-order passive RC low-pass filter. Fractional orders revealed temporal domain behaviour evolution. MATLAB output fractional-order low-pass filters. This function is approximated by the new concept of curve fitting. This method explains all kinds of fractional-order filters using a universal fractional-order transfer function. Work in [25] depicts a fractional-order element analogue modelling system. The classic circuit theory is used. Various procedures for the realization of the fractional element is discussed in [26]. Simulated frequency response changes fractionally. An active current mode fractional order analogue filter is designed using a single operational transresistance amplifier (OTRA). The proposed 1.5-order filter. PSPICE simulation confirmed theoretical results. The OTRA-made filter used RC-RC decomposition. The application of fractional order devices in the processing of bio-medical signals using Operational Transconductance Amplifiers is presented in [27] by P.Prommee et.al. An elaborate study of different developments in the field of realizing the fractance device is available in [28]. Different equivalent circuit representations of fractional order elements is presented in [29,30]. A fractional order bandpass filter realization using reconfigurable device is presented in [31].

The paper is organised as follows. Different Rational Approximation methods are discussed in Section 2. Section 3 deals with the fractional order lowpass filter. Section 4 discusses the implementation of the filter. The finalized circuit and the corresponding results are presented in section 5. Finally Conclusions are drawn in Section 6.

2. RATIONAL APPROXIMATION METHODS

Since the fractional order system is truly an infinite order system, approximation of finite length is preferred. The fractional order operator s^α can be approximated using both Laplace and z domains. Oustaloup method, Mastudas Method, Least square method etc. falls under s -domain approximations. Each method has its pros and cons. In this paper, two approximations named continued fraction expansion Method and Reyad El-Khazali's approximation are considered. Keeping in view of the hardware complexity only approximation upto second order is considered. The second order approximation of s^α is given as[4],

$$s^\alpha = \frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} \quad (1)$$

where a_0, a_1, a_2 are the filter coefficients.

2.1. Continued Fraction Expansion Method(CFE)

The continued fraction expansion of $(1+x)^\alpha$ is used to characterise this technique[4,32],

$$(1+x)^\alpha = \frac{1}{1-} \frac{\alpha x}{1+} \frac{(1+\alpha)x}{2+} \frac{(1-\alpha)x}{3+} \frac{(2+\alpha)x}{2+} \frac{(2-\alpha)x}{5+} \dots \quad (2)$$

When $x = s - 1$ is substituted, the above continuous fraction expansion converges down the negative real axis from $s = -\infty$ to $s = 0$ in the finite complex s -plane.

Considering the first four terms of the expansion a second order approximation will be derived with the coefficients as,

$$\begin{aligned} a_0 &= \alpha^2 + 3\alpha + 2 \\ a_1 &= 8 - 2\alpha^2 \\ a_2 &= \alpha^2 - 3\alpha + 2 \end{aligned} \quad (3)$$

2.2. Reyad El-Khazali's approximation (RE)

Reyad El-Khazali et al. [14] presented this approximation in 2019. A biquadratic approximation approach can be used to estimate a fractance operator. The rational approximation will take the shape of a cascade connection of biquadratic transfer functions and is as below,

$$\left(\frac{s}{\omega_g}\right)^\alpha = \prod_{i=1}^n H_i \left(\frac{s}{\omega_i}\right) = \prod_{i=1}^n \frac{N_i \left(\frac{s}{\omega_i/\omega_g}\right)}{D_i \left(\frac{s}{\omega_i/\omega_g}\right)}, \quad (4)$$

where

$$\begin{aligned} \omega_i &= \text{Center Frequency} \\ \omega_g &= \text{Geometric Mean} \end{aligned} \quad (5)$$

The remaining centre frequencies may be computed using a recursive formula using ω_1 ,

$$\omega_i = \omega_x^{2(i-1)} \omega_1, \quad i = 2, 3, 4, \dots, n. \quad (6)$$

ω_x is calculated by solving the equation,

$$a_0 a_2 \eta^4 + a_1 (a_2 - a_0) \eta^3 + (a_1^2 - a_2^2 - a_0^2) \eta^2 + a_1 (a_2 - a_0) \eta + a_0 a_2 \eta = 0, \quad (7)$$

where $\eta = \tan\left(\frac{\alpha\pi}{4}\right)$. The expression for the quadratic will be

$$\left(\frac{s}{\omega_g}\right)^\alpha \cong \frac{a_0 \left(\frac{s}{\omega_i}\right)^2 + a_1 \left(\frac{s}{\omega_i}\right) + a_2}{a_2 \left(\frac{s}{\omega_i}\right)^2 + a_1 \left(\frac{s}{\omega_i}\right) + a_0}, \quad i = 1, 2, 3, \dots \quad (8)$$

The values of the coefficients a_0, a_1, a_2 are calculated to be,

$$\begin{aligned} a_0 &= \alpha^\alpha + 2\alpha + 1 \\ a_2 &= \alpha^\alpha - 2\alpha + 1 \\ a_1 &= (a_2 - a_0) \tan\left(\frac{(2+\alpha)\pi}{4}\right). \end{aligned} \quad (9)$$

After calculating the coefficients, the rational approximation for s^α may be determined. The second order rational approximations obtained for different values of α is as shown in Table.1.

Table 1. Rational Approximation of second order

α	$H_{RE}(s)$	$H_{CFE}(s)$
0.1	$1.994s^2 + 5.082s + 1.594$	$2.31s^2 + 7.98s + 1.71$
	$1.594s^2 + 5.082s + 1.994$	$1.71s^2 + 7.98s + 2.31$
0.2	$2.125s^2 + 5.051s + 1.325$	$2.64s^2 + 7.92s + 1.44$
	$1.325s^2 + 5.051s + 2.125$	$1.44s^2 + 7.92s + 2.64$
0.3	$2.297s^2 + 4.998s + 1.097$	$2.99s^2 + 7.82s + 1.19$
	$1.097s^2 + 4.998s + 2.297$	$1.19s^2 + 7.82s + 2.99$
0.4	$2.493s^2 + 4.924s + 0.8931$	$3.36s^2 + 7.68s + 0.96$
	$0.8931s^2 + 4.924s + 2.493$	$0.96s^2 + 7.68s + 3.36$
0.5	$2.707s^2 + 4.828s + 0.7071$	$3.75s^2 + 7.5s + 0.75$
	$0.7071s^2 + 4.828s + 2.707$	$0.75s^2 + 7.5s + 3.75$
0.6	$2.936s^2 + 4.71s + 0.536$	$4.16s^2 + 7.28s + 0.56$
	$0.536s^2 + 4.71s + 2.936$	$0.56s^2 + 7.28s + 4.16$
0.7	$3.179s^2 + 4.569s + 0.3791$	$4.59s^2 + 7.02s + 0.39$
	$0.3791s^2 + 4.569s + 3.179$	$0.39s^2 + 7.02s + 4.59$
0.8	$3.437s^2 + 4.404s + 0.2365$	$5.04s^2 + 6.72s + 0.24$
	$0.2365s^2 + 4.404s + 3.437$	$0.24s^2 + 6.72s + 5.04$
0.9	$3.71s^2 + 4.215s + 0.1095$	$5.51s^2 + 6.38s + 0.11$
	$0.1095s^2 + 4.215s + 3.71$	$0.11s^2 + 6.38s + 5.51$

A comparison of the two rational approximations (for $\alpha = 0.4$) is as shown in Fig.1 and 2. The phase response offered by the RE approximation is good as compared to CFE based method. From the magnitude response it is observed that the CFE based method produces good results for larger range of frequencies as compared to RE method. Keeping in view of the better linearity in phase angle, RE method is considered for the implementation.

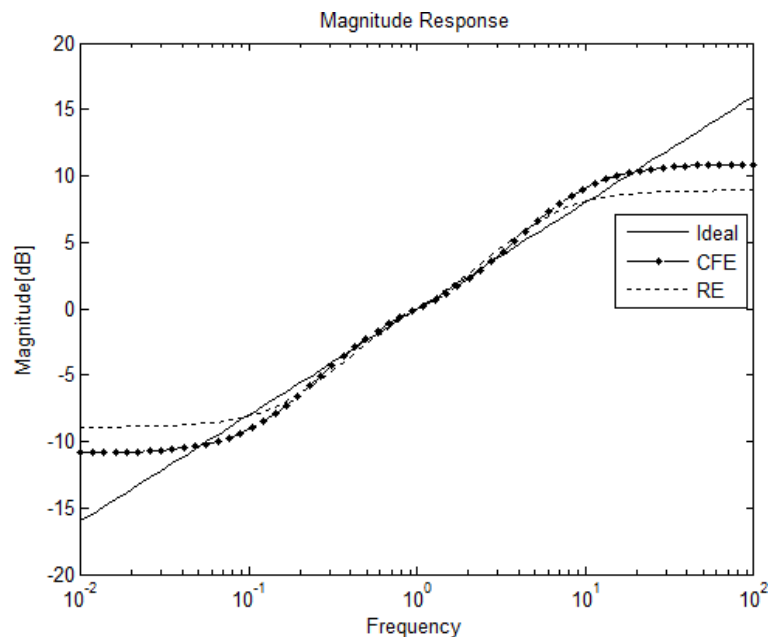


Figure 1. Comparison for Magnitude Response of different approximation methods

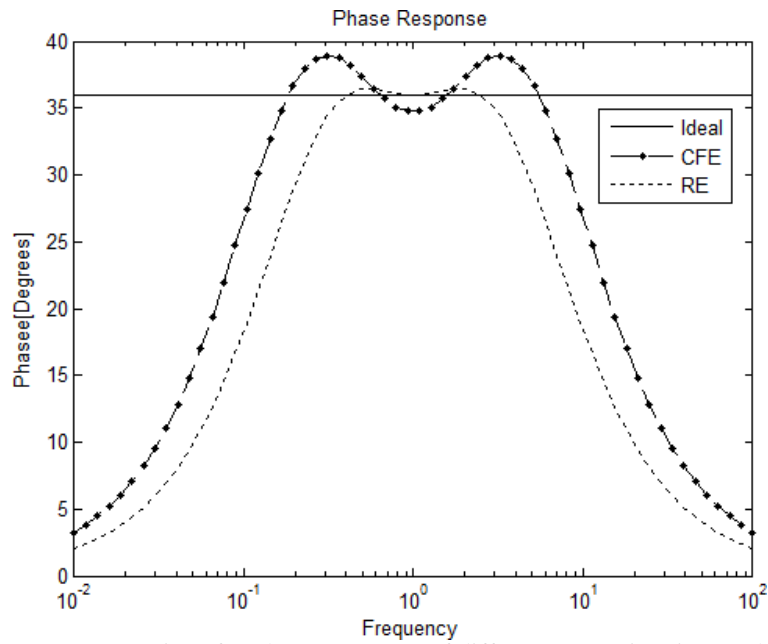


Figure 2. Comparison for Phase Response of different approximation methods

3. FRACTIONAL ORDER LOWPASS FILTER

Fractional order filters can meet the exact design criteria, but traditional integer-order filters were unable to do so. Comparing the design parameters to the integer-order filters, the fractional notion increases the level of safety a little amount. Fractional order filters make it simple to meet the requirements for the circuit design and tuning parameters. Only fractional order filters make it feasible to vary and modify parameters like the roll-off frequency to any desired slope. Thus, compared to integer-order filters, fractional order filters offer greater design freedom.

A standard filter and a fractional-order filter vary in that additional parameters are included in the term that determines the slope of attenuation of a given transfer function. Therefore, the equation for a fractional-order filter's attenuation of transfer function is $20(n + \alpha)$ (dB/decade), where n is an unsigned integer number, often between 1 and 10, and α is defined as a real number in the range $[0 \ 1]$. The transfer function of Fractional order low pass filter is given by[24],

$$H_{n+\alpha}^{LP} = \frac{k_1}{s^\alpha (s^n + k_2) + k_3} \quad (10)$$

where n is an integer and $0 < \alpha < 1$. The following are the values of k 's

$$\begin{aligned} k_1 &= 1 \\ k_2 &= 1.1796\alpha^2 + 0.167\alpha + 0.21735 \\ k_3 &= 0.19295\alpha + 0.81369 \end{aligned} \quad (11)$$

Substituting the second order approximation of the fractional order operator in the transfer function of the Lowpass filter, the equation becomes as,

$$H_{n+\alpha}^{LP} = \frac{k_1}{\left(\frac{a_0 s^2 + a_1 s + a_2}{a_2 s^2 + a_1 s + a_0} \right) (s^n + k_2) + k_3} \quad (12)$$

After rearranging the terms the equation becomes,

$$H_{n+\alpha}^{LP} = \frac{k_1 (a_2 s^2 + a_1 s + a_0)}{\left(\begin{aligned} &a_0 s^{n+2} + a_1 s^{n+1} + a_2 s^n + s^2 (k_3 a_2 + k_2 a_0) \\ &+ s (k_3 a_1 + k_2 a_1) + (k_3 a_0 + k_2 a_2) \end{aligned} \right)} \quad (13)$$

For $n = 1$, the above equation simplifies to,

$$H_{1+\alpha}^{LP} = \frac{k_1 (a_2 s^2 + a_1 s + a_0)}{a_0 (s^3 + c_1 s^2 + c_2 s + c_3)} \quad (14)$$

where,

$$\begin{aligned} c_1 &= (k_3 a_2 + k_2 a_0 + a_1) / a_0 \\ c_2 &= (k_3 a_1 + k_2 a_1 + a_2) / a_0 \\ c_3 &= (k_3 a_0 + k_2 a_2) / a_0 \end{aligned} \quad (15)$$

The equations indicate that the values of the filter coefficients are dependent upon the value of fractional order α . The following Table.2 depict the values of filter coefficients for different value of fractional order.

Table 2. Values of Filter co-efficients for different values of α

α	a_0	a_1	a_2	c_1	c_2	c_3
0.1	2.31	7.98	1.71	3.497325	0.9053559	0.04038944
0.2	2.64	7.92	1.44	3.101633	0.9029765	0.08254491
0.3	2.99	7.82	1.19	2.794686	0.9580753	0.1200770
0.4	3.36	7.68	0.96	2.563302	1.046208	0.1501903
0.5	3.75	7.5	0.75	2.397695	1.149750	0.1721550
0.6	4.16	7.28	0.56	2.29044	1.255711	0.1864237
0.7	4.59	7.02	0.39	2.235792	1.354332	0.1941091
0.8	5.04	6.72	0.24	2.229228	1.438158	0.1966716
0.9	5.51	6.38	0.11	2.267138	1.501410	0.1957304

The transfer functions of the FOLPF using different rational approximation techniques(For different values of α) is as Tabulated in Table.3.

Table 3. Transfer functions of Fractional order Lowpass Filter

α	$H_{LPCFE}(s)$	$H_{LPRE}(s)$
0.1	$\frac{1.71s^2 + 7.98s + 2.31}{2.31s^3 + 8.0788s^2 + 2.0914s + 0.0933}$	$\frac{1.594s^2 + 5.0820s + 1.9940}{1.9940s^3 + 5.1696s^2 + 1.8369s + 0.0839}$
0.2	$\frac{1.44s^2 + 7.92s + 2.64}{2.64s^3 + 8.1883s^2 + 2.3839s + 0.2179}$	$\frac{1.325s^2 + 5.051s + 2.1250}{2.1250s^3 + 5.2734s^2 + 1.9269s + 0.1888}$
0.3	$\frac{1.19s^2 + 7.82s + 2.99}{2.99s^3 + 8.3561s^2 + 2.8646s + 0.359}$	$\frac{1.097s^2 + 4.998s + 2.297}{2.297s^3 + 5.4204s^2 + 2.1973s + 0.3044}$
0.4	$\frac{0.96s^2 + 7.68s + 3.36}{3.36s^3 + 8.6127s^2 + 3.5153s + 0.5046}$	$\frac{8.931s^2 + 4.9240s + 2.493}{2.493s^3 + 5.63s^2 + 2.5314s + 0.4206}$
0.5	$\frac{0.75s^2 + 7.5s + 3.75}{3.75s^3 + 8.9914s^2 + 4.3116s + 0.6456}$	$\frac{7.071s^2 + 4.8280s + 2.707}{2.707s^3 + 5.9205s^2 + 2.9998s + 0.5287}$
0.6	$\frac{0.56s^2 + 7.28s + 4.16}{4.16s^3 + 9.5282s^2 + 5.2238s + 0.7755}$	$\frac{0.536s^2 + 4.71s + 2.9360}{2.9360s^3 + 6.3130s^2 + 3.5533s + 0.6212}$
0.7	$\frac{0.39s^2 + 7.02s + 4.59}{4.59s^3 + 10.2623s^2 + 6.2164s + 0.8910}$	$\frac{3.791s^2 + 4.569s + 3.179}{3.179s^3 + 6.8293s^2 + 4.1712s + 0.6928}$
0.8	$\frac{0.24s^2 + 6.72s + 5.04}{5.04s^3 + 11.2353s^2 + 7.2483s + 0.9912}$	$\frac{0.2365s^2 + 4.4040s + 3.437}{3.437s^3 + 7.4944s^2 + 4.8294s + 0.7407}$
0.9	$\frac{0.11s^2 + 6.38s + 5.51}{5.51s^3 + 12.4919s^2 + 8.2728s + 1.0785}$	$\frac{0.1095s^2 + 4.215s + 3.71}{3.71s^3 + 8.3364s^2 + 5.5023s + 0.7653}$

4. IMPLEMENTATION OF THE PROPOSED FILTER

In order to implement the transfer functions derived in the previous sections, follow the leader feed-back(FLF) topology is used. In this paper, feed-forward path based FLF structure is used for implementation. The outputs are available in terms of $s^n V_{out}$, $s^{n-1} V_{out}$,, $s V_{out}$, V_{out} . These outputs are used in a feed-forward form to get n^{th} order filter with arbitrarily transmission zeros as shown in Fig.3. In this structure, the outputs from the lossless integrators are multiplied by coefficients and then added together to get final output[34].

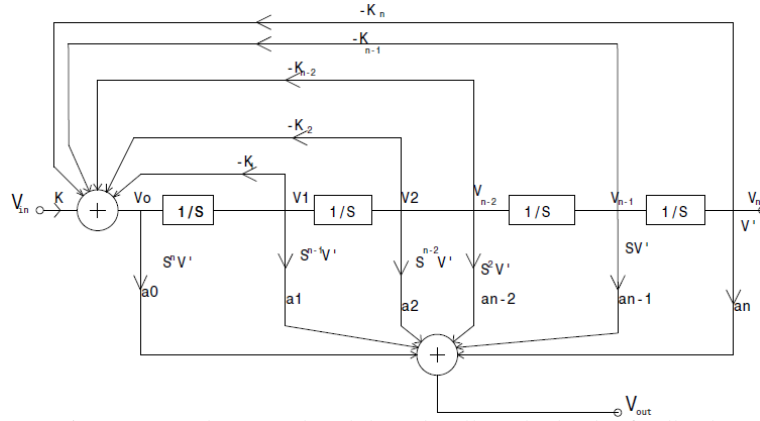


Figure 3. Feed Forward Path based Follow the leader feedback

$$\frac{V_1}{V_{in}} = H_1(s) = \frac{1}{s^n + k_1 s^{n-1} + k_2 s^{n-2} \dots k_{n-1} s + k_n} \quad (16)$$

$$\frac{V_{out}}{V_1} = H_2(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \dots a_{n-1} s + a_n \quad (17)$$

$$\begin{aligned} \frac{V_{out}}{V_{in}} &= H(s) = H_1(s)H_2(s) \\ &= \frac{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} \dots a_{n-1} s + a_n}{s^n + k_1 s^{n-1} + k_2 s^{n-2} \dots k_{n-1} s + k_n} \end{aligned} \quad (18)$$

The above transfer function can be realized using only lossless integrators and multipliers.

$$\frac{V_{out}}{V_{in}} = \frac{V_{out}}{V_1} \frac{V_1}{V_{in}} = H_1 H_2 = \frac{As^2 + Bs + C}{s^3 + Ds^2 + Es + F} \quad (19)$$

Now splitting the transfer function,

$$\begin{aligned} H_1 &= \frac{V_{out}}{V_1} = As^2 + Bs + C \\ \Rightarrow V_{out} + (-As^2 V_1 - Bs V_1 - CV_1) &= 0 \\ \Rightarrow V_{out} + (-s^2 V_1 \frac{1}{1/A} - s V_1 \frac{1}{1/B} - V_1 \frac{1}{1/C}) &= 0 \end{aligned} \quad (20)$$

$$H_2 = \frac{V_1}{V_{in}} = \frac{1}{s^3 + Ds^2 + Es + F} \quad (21)$$

The above equation can be rearranged as,

$$\begin{aligned} V_{in} - (s^3 + Es)V_1 - (Ds^2 + F)V_1 &= 0 \\ \Rightarrow V_{in} - \left(s^3 + s \frac{1}{1/E}\right) V_1 - \left(s^2 \frac{1}{1/D} + \frac{1}{1/F}\right) V_1 &= 0 \end{aligned} \quad (22)$$

The operational amplifier based realization valid for any fractional order α is as shown in Fig.4.

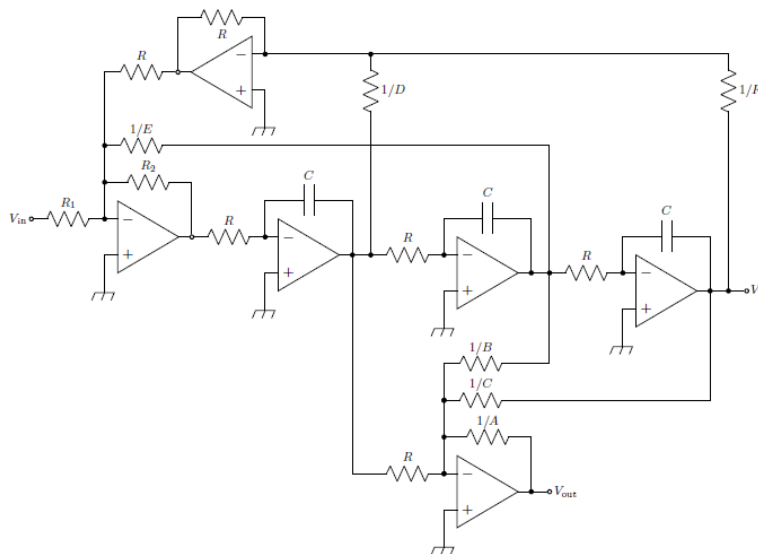


Figure 4. Synthesized Circuit for any value of α

5. RESULTS

To Illustrate the methodology, a value of $\alpha = 0.7$ is considered. The following is the transfer function of R-Elkhazalil Approximation is taken into consideration.

$$H_{1.7}^{LP}(s) = \frac{3.791s^2 + 4.569s + 3.179}{3.179s^3 + 6.8293s^2 + 4.1712s + 0.6928} \quad (23)$$

The circuit realized is as shown in Figure.5.

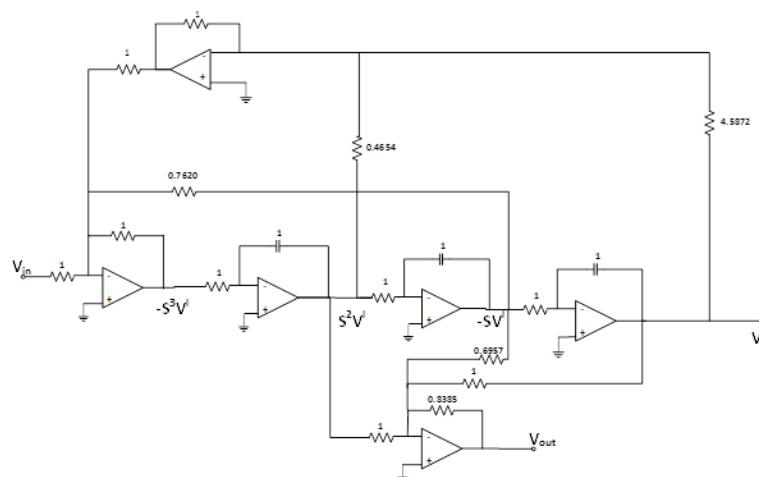


Figure 5. Realized Circuit for $\alpha = 0.7$ using R-Elkhalil Approximation

If Frequency denormalization by a factor of 20k rad/sec, and impedance scaling by a value of 10000, then all resistances and capacitance values will be practicable. The same circuit is used for the simulation purpose using TINA Software. For the simulation purpose Texas instruments TINA Software is used. The corresponding results of Transient Response, Bode plot are as shown in Fig.6 and 7 respectively.

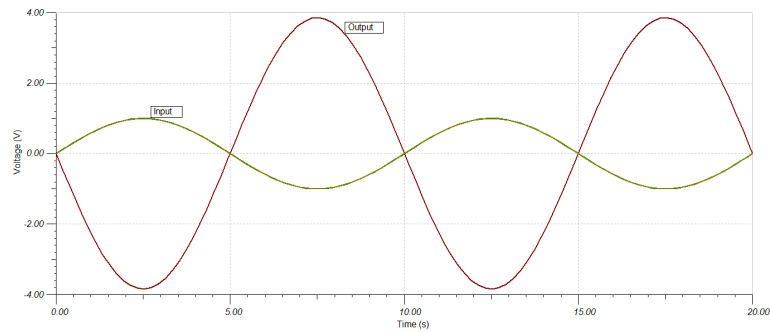


Figure 6. Transient Response

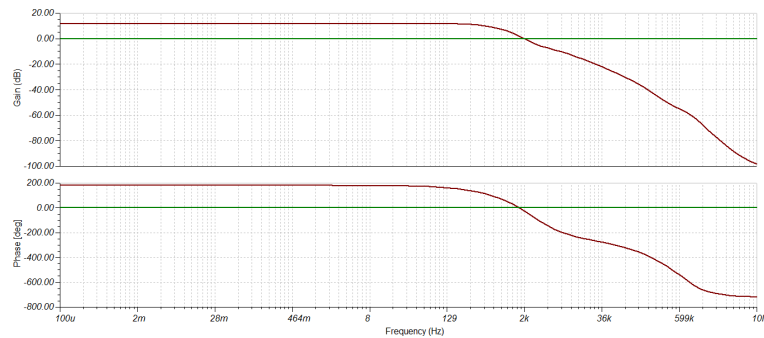


Figure 7. Bode Plot

6. CONCLUSIONS

This paper presents a novel realization of fractional order lowpass filter using different approximation techniques. Initially, a comparative study of the existing approximation techniques for fractional order device is carried out. Then synthesis of the fractional order lowpass filter is carried out using follow the leader feedback topology. The proposed method is straight forward and makes use of an operational amplifier for the realization. This method makes use of only lossless integrators and amplifiers. A 100mHz, $1V_{PP}$ Sinusoidal signal is chosen as input. LM301A operational amplifier is chosen for simulation purpose. It is observed that there is a phase shift of 180 between input and output. The linear range of operation can be upto $100\mu\text{Hz}$ to 2KHz. So, the procedure suggested can be used for the realization of fractional order filters.

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